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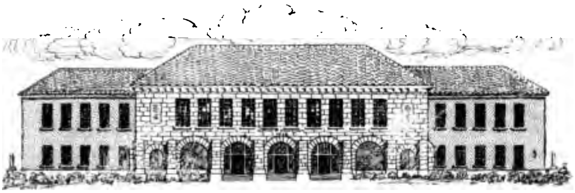

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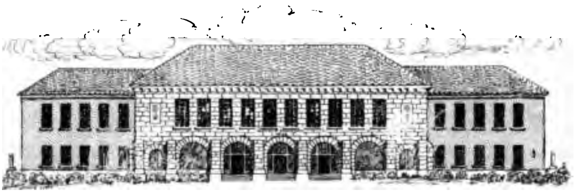

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A SUPPLEMENT

TO ACCOMPANY

THE ADVANCED ARITHMETIC

IN THE

CALIFORNIA STATE SERIES

BY

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Dedicated

TO THE BOYS AND GIRLS WHO THINK THE STUDY THAT
SHOULD BE THE EASIEST TO COMPREHEND,
THE MOST DIFFICULT

PREFACE.

THIS little book is based upon notes used, and work done, by the author, in the schoolroom with the several grades for which it is designed.

It is believed to be practical and logical. The purpose has been to make the statements clear, concise, and just as they would be told outside of a schoolroom, divorced from all the mystified atmosphere of something foreign to real life.

There seems to be a demand from teachers, parents, and pupils for something more explicit upon the different principles of arithmetic than is given in the State text-book. This demand, coupled with the author's own experience and success in the line of work herein suggested, together with the expressed desire of some of his co-laborers, induced him to put his notes into book form for the use of both pupils and teachers.

It is hoped that the work will meet with favor, and that some of the seemingly difficult things may be made easier by its use.

A. W. P.

LOS ANGELES, CAL.

SUPPLEMENT TO ADVANCED ARITHMETIC

TO ACCOMPANY THE

CALIFORNIA STATE SERIES.

ARITHMETIC.

Beyond the writing and reading of numbers — whole and decimal — and the principles of addition, subtraction, multiplication, and division, there is but little more than is involved in the questions of price, quantity, and cost, with which all school boys and girls are familiar.

The first year's work with the advanced arithmetic is the most important, as it lays, or should lay, the foundation of all that follows.

There should be less memorizing and more thinking.

There must be a unit or base upon which all operations are built.

All topics must be mastered, as each prepares for succeeding ones, and the reviews should be sufficiently frequent and comprehensive to fix the relations existing between the different parts of the whole subject. I trust that the topical outline at the end of this little work may be of value to teachers and pupils in making their reviews.

Arithmetic teaches the knowledge and use of numbers.

A **number** is a unit or a collection of units; as, 1, 27.

Every number has a unit of measure; as, 1 for 7, $1 for $7; and in 7 or $7 there are 7 of the units of measure.

A **unit** is a single thing, or one; as, 1, $1. (The first conception of numbers is obtained from their use in connection with objects.)

A **concrete** (joined with) **unit** is one joined to a name; as, 1 hour.

An **abstract** (taken from) **unit** is one without a name; as, 1.

A **concrete number** is one whose unit is concrete; as, 8 hours.

An **abstract number** is one whose unit is abstract; as, 8.

NOTATION.

Notation (noting or writing) is the writing of numbers.

Decimal (pertaining to ten) **notation** is that in which ten of any order equals one of the next higher. It is American notation.

A **decimal point** is a point (period) at the right and left of which numbers are written; the first figure on the left is units; 75., 7.5, .75.

A **figure** is a character used in representing numbers. There are ten figures; namely, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

A **digit** is a figure. (A figure is sometimes called a digit.)

The order of a figure is the position it occupies with regard to the decimal point. The first order on the left is units; the second, tens; the third, hundreds. On the right, the orders are tenths, hundredths, etc.

For each order on the left, excepting units, there is one with a corresponding name, on the right of the decimal point; as, 273 and .72. In the first number are 7 tens, and 7 tenths is the corresponding digit in the decimal; 2 hundreds, 2 hundredths. It requires one more figure to represent hundreds than it does to represent hundredths.

(The suffix *ty* is equivalent to tens; as, seventy, 7 tens; twenty, 2 tens.)

A group of three figures having a group name is a period. Each full period has three orders, units, tens, and hundreds of that period; as, 475,823, 3 units, 2 tens, and 8 hundreds of units' period; 5 units, 7 tens, and 4 hundreds of thousands' period.

The periods are units, thousands, millions, billions, etc.

A separating point is a comma placed between periods to aid in reading numbers; as, 27, 489, 567.

To write a number, begin at the left, write the figures of each period in their proper orders, place a separating point at the right of each period, as the number is written — for convenience, only — and fill all vacancies with ciphers. As 487 looks just the same in one period as in another, it is just as easy to put it into one as another. A pupil that really knows how to write the three figures of units' period can write any number as soon as he learns the positions of the periods with regard to the decimal point — the keynote.

NUMERATION.

Numeration (numbering) is the reading of numbers.

To read a number, begin at the right and point it off into periods — three figures each — with separating points; then begin at the left, read the number in each period, in order, giving the name of each period excepting that of units. The word *number* means a whole number, one on the left of the decimal point.

The **Roman notation** (developed by the Romans) uses seven capital letters; as, I for one, V for five, X for ten, L for fifty, C for one hundred, D for five hundred, and M for one thousand.

Different numbers are expressed by the following laws of combination:

I. Repeating a letter repeats its value; as, I, one; II, one and one, or two. (No letter repeats more than twice, one exception.)

II. Placing a letter before one of greater value subtracts its value from that of the greater; as, I, one; V, five; IV, five less one, or four.

III. Placing a letter after one of greater value adds its value to that of the greater; as, V, five; I, one; VI, five plus one, or six.

IV. Placing a letter between two of greater values subtracts its value from their sum; as, X, ten; V, five; I, one; XIV, ten and five less one, or fourteen.

V. Placing a dash over a letter multiplies its value by one thousand; as, X, ten; $\overline{X}$, one thousand times ten, or ten thousands.

ADDITION.

Addition is the finding of the sum of several like numbers,—numbers with like units.

The **sign** of addition is an erect cross (+), and is read "plus."

To add numbers, write them so that figures of the same order shall stand in the same column, and draw a horizontal line beneath; begin at the right and add each column separately, divide the several sums, except the last, by ten, write each remainder under the column of the same order, and add each quotient to the column of the next higher order. Write the entire sum of the left column.

$$\begin{array}{r} 349 \\ 856 \\ 437 \\ \hline 1642 \end{array}$$

ILLUSTRATED: 7, 13, 22 units, which I divide by ten as it takes 10 units to make 1 ten, giving 2 tens and 2 units. Write 2 in units' place in the sum and add 2 tens to the column of tens. 2, 5, 10, 14 tens, which I divide by ten as it takes 10 tens to make 1 hundred, giving 1 *h* and 4 *t*. Write 4 in tens' place in the sum, and add 1 *h* to the column of hundreds. 1, 5, 13, 16 hundreds, which I divide by ten, as it takes 10 *h* to make 1 thousand, giving 1 thousand and 6 *h*. Write 6 in hundreds' place, and 1 in thousands' place in the sum. The sum is 1642. (Observe the same analysis in addition of compound denominate numbers.)

Prove addition by adding downward, by casting out the tens, or by casting out the nines. To cast out the tens, add as usual until the sum is 10 or more, but less than 20, make a dash beside the last figure added, for the 10, and continue with the unit figure. After completing the column, write the unit figure and add as many tens of that column as there are dashes, to the next column; as, 7, 13, — (beside 6), 3, 12, — (beside 9), 2. Place 2 in the answer and add 2 to the next column, and thus continue.

The sum of the excess of 9's in the sum of the digits of the several numbers equals the excess of 9's in the sum of the digits in the answer; as, in above, 3, 4, 9 give one 9 and excess of 7; 8, 5, 6 give two 9's and excess of 1; 4, 3, 7 give one 9 and excess of 5; excesses 7, 1, 5 give one 9 and excess of 4; 1, 6, 4, and 2 give one 9 and excess of 4. Excesses are even.

SUBTRACTION.

Subtraction (take from) is the taking of one number from another of the same kind.

The **terms** of subtraction are minuend (m), subtrahend (s), and remainder (r).

The **minuend** is the number to be diminished; the **subtrahend** is the number to be taken from the minuend; and the **remainder** is the part of the minuend to be left.

The **sign** of subtraction is a short horizontal line ($-$), and is read "minus" or "less."

The sign of equality is two short horizontal lines ($=$) and is read equals or is equal to; $9 - 5 = 4$, 9 minus 5 equals 4.

A **formula** is a little form showing how to do many examples.

The formulas for subtraction are: $m - s = r$, $11 - 7 = 4$; $m - r = s$, $11 - 4 = 7$; $s + r = m$, $7 + 4 = 11$.

To subtract, place the subtrahend under the minuend (for convenience) so that units of the same order, or the decimal points, shall stand under each other, and draw a horizontal line beneath. Begin at the right and subtract each figure of the subtrahend from the one of the same order in the minuend, and place each remainder under the order subtracted.

Should any figure of the minuend be smaller than the one of the same order of the subtrahend, reduce one of the next higher order to units of this order, and add it to said figure of the minuend before subtracting. Observe, in subtracting the next figure, that the figure of the minuend has been diminished by one.

$$\begin{array}{r}2358\\1276\\\hline 1082\end{array}$$

ILLUSTRATED: 6 *u* from 8 *u* leaves 2 *u*, which I write in units' place in the remainder. 7 *t* I cannot take from 5 *t*, so I take 1 *h* from 3 *h*, leaving 2 *h*. 1 *h* equals 10 *t*; 10 tens plus 5 tens equals 15 tens. 15 *t* minus 7 *t* equals 8 *t*, which I write in tens' place in the remainder. 2 *h* from 2 *h* leaves no hundreds. I write a cipher in the hundreds' place in the remainder. 1 *th* from 2 *th* leaves 1 *th*, which I write in the thousands' place. My answer is 1082.

Subtraction is the reverse of addition. It depends upon addition: 9 less 6 equals 3, because 3 added to 6 makes 9.

The proofs of subtraction are indicated by the formulas.

GENERAL FORM: What is the difference between — and —?

MULTIPLICATION.

Multiplication is the taking of one number as many times, additively, as there are units in another. It is contracted addition: 3 times 6 means 6 plus 6 plus 6, or 18.

The **terms** of multiplication are multiplier (*mr*), multiplicand (*md*), and product (*pd*). The **sign** of multiplication is an oblique cross (×), and is read "times" or "multiplied by."

The *md* is the number taken; the *mr* is the number showing how many times the *md* is to be taken; the *pd* is the result obtained.

The *md* and *mr* are sometimes called **factors.**

The formulas are: $mr \times md = pd$, $3 \times 7 = 21$; $pd \div md = mr$, $21 \div 7 = 3$; $pd \div mr = md$, $21 \div 3 = 7$.

To multiply, write the *mr* under the *md*, and draw a line beneath. Begin at the right and multiply each figure of the *md* by each figure of the *mr*; divide each individual product, in order, by 10 and write the remainder in its order in the partial product and add the quotient to the next individual product, and thus proceed to the last individual product.

The sum of the partial products gives the complete product.

$$\begin{array}{r} 497 \\ 5 \\ \hline 2485 \end{array}$$

Illustrated: 5 times 7 units are 35 units, which I divide by 10, as it takes 10 units to make one ten, giving 3 *t* and 5 *u*. I write 5 in units' place in the product and add 3 *t* to the product of tens. 5 times 9 *t* are 45 *t*, 45 with 3 *t* equals 48 *t*, which I divide by 10, as it takes 10 tens to make 1 *h*, giving 4 *h* and 8 *t*. I write 8 in tens' place in the product and add 4 *h* to the product of hundreds. 5 times 4 *h* are 20 *h*, 20 *h* with 4 *h* equals 24 *h*, which I divide by 10, as it takes 10 hundreds to make 1 *th*, giving 2 *th* and 4 *h*. I write 4 in the hundreds' place in the product, and, as there are no thousands to be multiplied, I write 2 in the thousands' place in the product. The product is 2485.

If the *mr* consist of more than one figure, multiply by each figure and place the right-hand figure of each product under the figure of the *mr* used; then take the sum of the partial products to find the complete product. In the analysis, each partial multiplier should be reduced to units; as, 437 is made of 400, 30, and 7 units.

The proofs are indicated by the formulas.

The name of the product is determined by the analysis, the processes of multiplication and division being with abstract numbers.

General Form: Find the product of — and —.

The principles of multiplication are:

I.
$$\begin{array}{rrrrr} 4\times & 6 & = & 24 \\ & \times\ 2 & & \times\ 2 \\ \cline{2-2}\cline{4-4} 4\times & 12 & = & 48 \end{array}$$

Multiplying the *md* by any number multiplies the *pd* by an equal number.

II.
$$\begin{array}{rrrr} 4\times & 6 & = & 24 \\ \times 2 & & & \times\ 2 \\ \cline{1-1}\cline{4-4} 8\times & 6 & = & 48 \end{array}$$

Multiplying the *mr* by any number multiplies the product by an equal number.

III.

$$\begin{array}{rrr} 4\times & 6= & 24 \\ \times 2 & \times\ 2 & \times\ 4 \\ \hline 8\times & 12= & 96 \end{array}$$

Multiplying both *md* and *mr* multiplies the *pd* by the product of the secondary (new) multipliers.

IV.

$$\begin{array}{rrr} 4\times & 6= & 24 \\ & \div\ 2 & \div\ 2 \\ \cline{2-3} 4\times & 3= & 12 \end{array}$$

Dividing the *md* by any number divides the *pd* by an equal number.

V.

$$\begin{array}{rrr} 4\times & 6= & 24 \\ \div 2 & & \div\ 2 \\ \cline{1-1}\cline{3-3} 2\times & 6= & 12 \end{array}$$

Dividing the *mr* by any number divides the *pd* by an equal number.

VI.

$$\begin{array}{rrr} 4\times & 6= & 24 \\ \div 2 & \div\ 2 & \div\ 4 \\ \hline 2\times & 3= & 6 \end{array}$$

Dividing both *md* and *mr* by any numbers divides the *pd* by the product of the divisors.

DIVISION.

Division is the finding of how many times one number is contained in another. It is the reverse of multiplication, or short addition, therefore it is short subtraction. 24 divided by 6 says, "How many 6's can be taken out of 24?"

(Addition, subtraction, multiplication, and division are the fundamental principles of arithmetic, and the last three are modifications of the first, addition.)

The **terms** of division are dividend (*dd*), divisor (*dr*), quotient (*quo*), and remainder (*r*), sometimes.

The *dd* is the number to be divided; the *dr* is the number by which to divide; the *quo* is the number showing how many times the *dr* is contained in the *dd*; and the *r*, if there be any, is the part of the *dd* left.

The signs of division are a short, curved line,), between *dr* and *dd*; a short horizontal line with a dot above and below it, ÷, between the *dd* and *dr*; a horizontal line with the *dd* above and the *dr* below it; and a colon between *dd* and *dr*; as, 6)18, 18 ÷ 6, $\frac{18}{6}$, 18 : 6.

To divide, for convenience, place the *dr* at the left of the *dd* with a right curve between them, and the *quo* at the right with a left curve between it and the *dd*, or place the *quo* above the *dd*, as it is obtained.

Find how many times the *dr* is contained in the fewest left-hand figures of the *dd* that will contain it. Place this number as the first figure of the *quo*, multiply the *dr* by this figure and subtract the product from the part of the *dd* used; to this remainder annex the next figure of the *dd*, obtaining a new partial *dd*. Continue in this manner until all of the *dd* has been used. If there be a remainder, it may be placed above the *dr* as a fractional part in the quotient.

"To this remainder annex the next figure of the *dd*" is equivalent to multiplying the remainder by 10 to reduce it to the next lower order and adding the figure of that order, the same principle that is used in compound denominate numbers.

Dr *Dd* *Quo*
$19)3786(199\frac{5}{19}$

ILLUSTRATED: 19 is contained in 37 *h*, 1 *h* times. Write 1 in the hundreds' place in the *quo* and multiply 19 by 1. 1 *h* times 19 are 19 *h*, and 19 *h* from 37 *h* leaves 18 *h*. Multiply 18 *h* by 10, as there are 10 times as many tens as hundreds, giving 180 *t*, which with 8 *t* make 188 *t*. 19 in 188 *t*, 9 *t* times. Write 9 in tens' place in the *quo*, and multiply 19 by it. 9 *t* times 19 are 171 *t*. 171 *t* from 188 *t* leaves 17 *t*. Multiply 17 *t* by 10, as there are 10 times as many units as tens, giving 170 *u*, which with the 6 *u* added, make 176 *u*. 19 is contained in 176 *u*, 9 *u* times. Write 9 in units' place in the *quo*. 9 *u* times 19 are 171 *u*. 171 *u* from 176 *u* leaves 5 *u*, of which, indicate the division by the fractional form, and place it as a part of a unit in the *quo*. (5 has been divided by 19 as truly as has the 171 *u* out of the 176.)

In dividing 23 by 5, 20 is divided by 5, giving 4; then 3 is divided by 5, giving $\frac{3}{5}$. It is no more difficult to divide 3 by 5 than it is to divide 20 by 5.

The work of short division is the same as that of long, excepting that it is done mentally and the work is not written.

The proofs of division are indicated by the formulas: $dd \div dr = quo$, $12 \div 3 = 4$; $quo \times dr = dd$, $4 \times 3 = 12$; $dd \div quo = dr$, $12 \div 4 = 3$.

The name of the quotient is obtained by the analysis.

General Form: How many times is — contained in — ? (Never say "goes into.")

$$\begin{array}{lrrr} & \mathit{Dd} & \mathit{Dr} & \mathit{Quo} \\ \text{I.} & 36 \div & 6 = & 6 \\ & \underline{\times\ 2} & & \underline{\times\ 2} \\ & 72 \div & 6 = & 12 \end{array}$$

Principles of Division: Multiplying the *dd* by any number multiplies the *quo* by an equal number.

$$\begin{array}{lrrr} \text{II.} & 36 \div & 6 = & 6 \\ & & \underline{\times\ 2} = & \underline{\div\ 2} \\ & 36 \div & 12 = & 3 \end{array}$$

Multiplying the *dr* by any number divides the *quo* by an equal number.

$$\begin{array}{lrrr} \text{III.} & 36 \div & 6 = & 6 \\ & \underline{\times\ 2} & \underline{\times\ 2} & \\ & 72 \div & 12 = & 6 \end{array}$$

Multiplying both *dd* and *dr* by equal numbers does not change the quotient.

$$\begin{array}{lrrr} \text{IV.} & 36 \div & 6 = & 6 \\ & \underline{\div\ 2} & & \underline{\div\ 2} \\ & 18 \div & 6 = & 3 \end{array}$$

Dividing the *dd* by any number divides the *quo* by an equal number.

$$\begin{array}{lrrr} \text{V.} & 36 \div & 6 = & 6 \\ & & \underline{\div\ 2} = & \underline{\times\ 2} \\ & 36 \div & 3 = & 12 \end{array}$$

Dividing the *dr* by any number multiplies the *quo* by an equal number.

$$\begin{array}{lrrr} \text{VI.} & 36 \div & 6 = & 6 \\ & \underline{\div\ 2} & \underline{\div\ 2} & \\ & 18 \div & 3 = & 6 \end{array}$$

Dividing both *dd* and *dr* by equal numbers does not change the quotient.

Condensed. Multiplying the *dd* or dividing the *dr* by any number multiplies the *quo* by an equal number. Dividing the *dd* or multiplying the *dr* by any number divides the *quo* by an equal number. Multiplying or dividing both *dd* and *dr* by equal numbers does not change the *quo.*

The work of fractions depends upon the principles of division.

Simple (because familiar) questions involving price, quantity, and cost may be used to illustrate the four fundamental principles. The business terms are price (p), quantity (q), cost (c), gain (g), loss (l), and selling price (sp). $c + g = sp$, $21 + 4 = 25$; $sp - c = g$, $25 - 4 = 21$; $p \times q = c$, $3 \times 7 = 21$; $c \div p = q$, $21 \div 3 = 7$, $c \div q = p$, $21 \div 7 = 3$. Make many other illustrations.

The analysis of this class of questions, with the previous work, exemplifies nearly all the operations of arithmetic.

Always make the analysis conform to the work, bearing in mind that dollars may be bought and sold.

Find cost of 375 barrels of apples at $ 2 per barrel.

Since 1 barrel cost $ 2, there are 2 times as many dollars as barrels, or 2 times 375, 750, or $ 750, cost.

Find cost of 2 house lots at $ 375 per lot.

Since 1 lot cost $ 375, 2 lots cost 2 times as many dollars, or 2 times 375, 750, or $ 750, cost.

One of the above forms of analysis is as logical as the other; use the one that gives the simpler operation.

Do not make false statements; as, $6 \times 3 = 18 \div 9 = 2$.

PROPERTIES OF NUMBERS.

The **property** of any person or thing is what belongs to that person or thing.

An **integer** is a whole number; 1, 23.

A **factor** is an exact integral divisor; as, 3 is a factor of 9. Why?

A **prime** number has no factor; as, 7, 13.

A **composite** number has factors; as, 8, 24. (Prime, first; composite, composed.)

Numbers are prime to each other if they have no like factor; as, 8, 11.

Two composite numbers may be prime to each other; as, 8, 15.

An **odd** number cannot be divided by 2; it may be prime or composite; as, 7, 9.

An **even** number can be divided by 2, and it includes every number whose right-hand figure is even; as, 2, 18, 20.

A **prime factor** is a prime number. 3 is a prime factor of 12 because it is a factor and is a prime number.

3 is a divisor of all numbers the sum of whose digits is divisible by 3; as, 768. The sum of 7, 6, and 8 is 21. 21 is divisible by 3, and the number 768 is. The explanation may be too difficult for fifth grade pupils. 768 is made up of 700 *u*, 60 *u*, and 8 *u*.

$$\begin{aligned} 700 &= 7 \times 100 = 7\,(99 + 1) = 7 \times 99 + 7 \\ 60 &= 6 \times 10 = 6 \times (9 + 1) = 6 \times 9 + 6 \\ 8 &= 8 \end{aligned}$$

The sum of 7, 6, and 8 is 21.

Any number is made up of a number of 9's and the sum of its digits; 3 is a divisor of 9 or any number of 9's, and if it be a divisor of the sum of the digits, it is a divisor of the number.

A **divisor** of two or more numbers is a divisor of their sum or difference; as, a divisor of 9 and 21 is a divisor of their sum (30), or their difference (12).

5 is a divisor of any number whose unit figure is 5 or 0; as, 75, 90. As the remainder next to the unit figure, obtained by dividing by 5, is a ten or some number of tens, and 5 is a divisor of ten or any number of tens, and as it is a divisor of the unit figure, 5 or 0, it is a divisor of the sum of the tens and units, and the whole number.

To find the prime factors of any number, divide it by any prime factor, its quotient by any other prime factor, and thus continue with each succeeding quotient until a quotient is obtained that is a prime number.

The divisors and the last quotient are the prime factors. Illustrate.

A **common divisor** of two or more numbers will divide each of them; as, 3, of 12, 15, and 21.

The **greatest common divisor** of two or more numbers is the greatest number that will divide each of them; as, 9 is the greatest common divisor of 18, 36, and 45.

The g. c. f. contains only the common prime factors, the least number of times they occur in any of the numbers; as, in 18, 36, and 45, 3 is the only common prime factor, and the least number of times that it occurs is two; therefore, 3 × 3, or 9, is the g. c. f.

Find the g. c. f. by inspection (looking and thinking) when possible.

Aids for Inspection: The g. c. f. cannot be greater than the smallest number; if not that number, it must be some factor of it. Try the largest factor. If any two of the numbers are prime to each other, the g. c. f. is 1.

ILLUSTRATED:	Case I.	4	Case II.	8	Case III.	8
		12		12		9
		16		16		16
	G. c. f.	4	G. c. f.	4	G. c. f.	1

I. G. c. f. equals 4, the smallest number. II. G. c. f. equals 4, the largest factor of the smallest number. III. G. c. f. is 1, as 8 and 9 are prime to each other.

In finding the g. c. f. of large numbers, divide the greater by the smaller, and the last divisor by the last remainder, and so on until there is no remainder. The last divisor is the g. c. f., because it is the greatest number that will divide the differences and the numbers.

ILLUSTRATED:	371		636
	265	1	371
	106	1	265
	106	2	212
		2	53, g. c. f.

The **greatest common factor** is used in cancellation and in reducing fractions to their lowest terms.

General Form: Find the g. c. f. of —, —, —.

MULTIPLE OR DIVIDEND.

A **multiple** is a number of times some other number, or will contain some other number. A multiple of a number is a dividend and can be divided by the number; as, 27 is a dividend of 9.

A **common multiple** of several numbers can be divided by each of them; as, 24 is a common multiple of 6 and 4.

The **least common multiple** (l. c. m.) of several numbers is the least number that can be divided by each of them; as, 24 is the l. c. m. of 6, 8, and 12.

The l. c. m. must contain all the different prime factors the greatest number of times they occur in any of the numbers; as, in 12, 16, and 18, the prime factors are 2 and 3. The greatest number of times that 2 occurs in either of them is 4; and 3 occurs 2 times. 2 × 2 × 2 × 2 × 3 × 3, or 144, is the l.c. m.

Find the l. c. m., when possible, by inspection.

Aids for Inspection: The l. c. m. can never be smaller than the largest number; if it be not that number, it must be some number of times that number. If the numbers be prime to one another, the l. c. m is their product.

Illustrated: Case I.	6	Case II.	4	Case III.	8
	8		8		9
	24		12		11
L. c. m.	24	L. c. m.	24	L. c. m.	792

I. L. c. m. is 24, the largest number. II. L. c. m. is 24, two times the largest number. III. L. c. m. is 792, the product of the numbers.

Case II. may be worked as follows:

$$\begin{array}{l} \cancel{4} \\ \cancel{8}\ 2 \\ \underline{12} \\ 24 \end{array}$$

Any number that will contain 12, will contain 4 or the factor, 4, in 8, and these may be suppressed (cut out), leaving 2 times 12, or 24.

The numbers may be arranged upon a horizontal line and divided by any prime number that will divide two or more of them; and the quotients and undivided numbers, if there be any, by another prime number that will divide two or more of them, and thus continue until the quotients and undivided numbers are prime to one another. The product of the divisors, last quotients, and undivided numbers is the l. c. m., because it is the product of all the different prime factors the greatest number of times they occur in either number.

Illustrated:

(1)

$$\begin{array}{r|rrrr} 2 & 12 & 16 & 24 & 36 \\ \hline 2 & 6 & 8 & 12 & 18 \\ \hline 2 & 3 & 4 & 6 & 9 \\ \hline 3 & 3 & 2 & 3 & 9 \\ \hline & 1 & 2 & 1 & 3 \end{array}$$

(2)

$$\left.\begin{array}{l} 12 = 2 \times 2 \times 3 \\ 16 = 2 \times 2 \times 2 \times 2 \\ 24 = 2 \times 2 \times 2 \times 3 \\ 36 = 2 \times 2 \times 3 \times 3 \end{array}\right\}\text{Prime factors.}$$

$2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$ l. c. m. The greatest number of times that 2 occurs as a factor in any of the numbers is four; 3 occurs twice; therefore, the l. c. m. is $2^4 \times 3^2$, or 144.

Ex. 1 shows a short process of finding the prime factors. Study it.

This principle is used in finding the least common denominator of fractions.

General Form: Find the l. c. m. of 12, 16, 24, and 36.

CANCELLATION.

Cancellation implies the act of cutting out.

When the product of several numbers is to be divided by the product of several other numbers, the factors of the first product may be arranged upon a horizontal line with the sign of multiplication between them, and the factors of the latter similarly arranged below the line.

Then the common factors of *dd* and *dr* may be cast out, after which, divide the product of the remaining factors above the line (*dd*) by the product of those below the line (*dr*).

ILLUSTRATED:

$$\frac{\overset{1}{\cancel{8}} \times \overset{2}{\cancel{12}} \times \overset{8}{\cancel{32}}}{\underset{1}{\cancel{6}} \times \underset{1}{\cancel{4}} \times \underset{3}{\cancel{24}}} = \frac{16}{3} \text{ or } 5\tfrac{1}{3}.$$

Divide 8 × 12 × 32 by 6 × 4 × 24.

Cancel the common factor, 8, from 8 and 24; 6, from 6 and 12; 4, from 4 and 32. Divide the product of 1, 2, and 8 by the product of 1, 1, and 3, giving $5\frac{1}{3}$.

This contracted process is very useful in multiplication of fractions, in general analysis, and in proportion.

FRACTIONS.

A **fraction** is an expression of division, with the *dd* above the *dr* and a line between them; as, $\frac{3}{7}$. It also expresses one or more equal parts of a unit; as, $\frac{3}{7}$ indicates that the unit has been divided into 7 equal parts and 3 of them taken.

The **terms** of a fraction are numerator and denominator.

The **numerator** (numberer) is written above the line, shows how many parts are taken, and corresponds to the dividend.

The **denominator** (namer) is written below the line, names the size of the parts, and corresponds to the divisor.

The value of a fraction is found by dividing the numerator by the denominator; as, $\frac{15}{5}$ equals 15 ÷ 5, or 3. It is a question of simple division, asking how many times the denominator is contained in the numerator.

A **unit** of a fraction is one of the number divided; as, \$1 is the unit of \$$\frac{5}{6}$.

A **fractional unit** is one of the equal parts; as, \$$\frac{1}{6}$ is the fractional unit of \$ $\frac{5}{8}$. A fractional unit is always a fraction whose numerator is 1.

The size of a fractional unit depends upon the size of the denominator. The larger the denominator, the smaller the fractional unit. Show it.

A **mixed number** is a combination of an integer and a fraction; as, $7\frac{2}{3}$.

A **compound fraction** is a combination of two or more simple fractions connected by *of;* as, $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{7}{8}$.

A **complex fraction** has a fraction in one or both of its terms; as, $\frac{7\frac{1}{2}}{\frac{5}{6}}$, $\frac{3}{8\frac{2}{7}}$.

Reduction of fractions is the changing of form without changing the value.

The **principles** of fractions are the same as those of division. Give them, using *num, denom,* and *val,* instead of *dd, dr,* and *quo.*

REDUCTION OF FRACTIONS.

To lower terms: divide both terms by a common factor or the g. c. f.

$\frac{4)\,8}{4)\,12} = \frac{2}{3}$ Principle: Dividing both *dd* and *dr* by equal numbers, etc.

To higher terms (common or l. c. denom.): multiply both terms by equal numbers; as, $\frac{7}{8}$ to 24ths. $\frac{7 \times 3}{8 \times 3} = \frac{21}{24}$.

Divide 24 by 8 to find the number by which 8 is multiplied to produce 24, then multiply the numerator (7) by an equal number (3).

Principle: Multiplying both *dd* and *dr* by equal numbers, etc.

To reduce fractions to their least common denominator (like fractions), find the l. c. m. of their denominators for a new denominator. Divide this denominator by each given denominator to find by what number each is multiplied, and multiply each numerator by the quotient obtained by its denominator.

ILLUSTRATED: $\frac{2}{3}\ \frac{3}{4}\ \frac{5}{6}$

$$\begin{array}{r|cccccc}
\text{Denoms.} & \text{L. c. m.} & & \text{Nums.} & & \text{New nums.} \\
 & 12 & & & & \\
\hline
3 & 4 & \times & 2 & = & 8 \\
4 & 3 & \times & 3 & = & 9 \\
6 & 2 & \times & 5 & = & 10
\end{array}
\qquad \frac{8}{12}\ \frac{9}{12}\ \frac{10}{12}.$$

L. c. m. of 3, 4, and 6 is 12.

To reduce a mixed number to a fraction, multiply it by a number equal to the denominator of the fractional part, or its multiple, and to keep its value, place a number equal to the multiplier under the product, as a denominator. Multiplying and dividing by equal numbers does not change the value; as, $7\frac{3}{4}$ multiplied by 4 equals 31; 31 divided by 4 may be expressed as $\frac{31}{4}$.

To reduce a fraction to a whole or mixed number, perform the division indicated.

Only like fractions can be added or subtracted. \$5 and \$2 are \$7; 5 ninths and 2 ninths are 7 ninths; $\frac{5}{9}$ and $\frac{2}{9}$ are $\frac{7}{9}$. \$5 less \$2 are \$3; 5 ninths less 2 ninths are 3 ninths; $\frac{5}{9}$ less $\frac{2}{9}$ are $\frac{3}{9}$. Add or subtract numerators, only, of like fractions.

To add mixed numbers, add the fractions, then the integers.

ILLUSTRATED: The l. c. d. of the fractional parts is written below the line and a little to the right. The new numerators are written in small figures, at the right of the given numerators, then proceed as in the illustration.

$$\begin{array}{rl}
18\frac{2}{3} & {}^{16} \\
17\frac{3}{4} & {}^{18} \\
9\frac{7}{8} & {}^{21} \\
\hline
\multicolumn{2}{l}{46\frac{7}{24}\ (\frac{55}{24} = 2\frac{7}{24}}
\end{array}$$

Four cases of subtraction of mixed numbers illustrated:

$$\text{I.}\ \begin{array}{r}287\frac{3}{4} \\ 196 \\ \hline 91\frac{3}{4}\end{array}
\qquad
\text{II.}\ \begin{array}{r}287 \\ 196\frac{3}{4} \\ \hline 90\frac{1}{4}\end{array}
\qquad
\text{III.}\ \begin{array}{rl}287\frac{3}{4} & {}^{9} \\ 196\frac{2}{3} & {}^{8} \\ \hline \multicolumn{2}{c}{91\frac{1}{12}\ (\frac{1}{12})}\end{array}
\qquad
\text{IV.}\ \begin{array}{rl}287\frac{2}{3} & {}^{8} \\ 196\frac{3}{4} & {}^{9} \\ \hline \multicolumn{2}{c}{90\frac{11}{12}\ (\frac{11}{12})}\end{array}$$

The fraction in the difference, (1) is the same as in the *m*; (2) the denominator is the same as that of the *s*, and the numerator is equal to the difference of the terms; (3) is difference of new numerators over the l. c. d.; (4) is equal to the new numerator of the *m* plus the l. c. d., less the new numerator of the *s*, over the l. c. d. 1 unit equals $\frac{12}{12}$, which added to $\frac{8}{12}$ makes $\frac{20}{12}$, $\frac{20}{12}$ less $\frac{9}{12}$ equals $\frac{11}{12}$.

Addition and subtraction of fractional units illustrated:

Sum of numerators over their product; $\frac{1}{5}$ plus $\frac{1}{6}$,

$$\frac{1}{5}+\frac{1}{6}=\frac{6+5}{5\times 6}=\frac{11}{30}.$$

Difference of numerators over their product; $\frac{1}{5}$ less $\frac{1}{6}$,

$$\frac{1}{5}-\frac{1}{6}=\frac{6-5}{5\times 6}=\frac{1}{30}.$$

(Figures of fractions should be made smaller than the integral figures, and horizontal lines placed between the terms, if the fractions be used in connection with integers.)

MULTIPLICATION OF FRACTIONS.

I. To multiply a fraction by an integer, multiply the numerator or divide the denominator of the fraction by the integer; as, $\frac{3}{8}\times 2$.

PRIN. Multiplying the *dd* (num.) or dividing the *dr* (denom.), etc.

$$\frac{3}{8}\times 2=\frac{3\times 2}{8}=\frac{6}{8}\text{ or }\frac{3}{4}.$$

$$\frac{3}{8}\times 2=\frac{3}{8\div 2}=\frac{3}{4}.$$

II. To multiply an integer by a fraction, multiply the integer by the numerator of the fraction and divide this product by the denominator; as, 8 multiplied by $\frac{3}{5}$. 3 times 8 is 24, 24 divided by 5 equals $\frac{24}{5}$. 3 is 5 times as large as $\frac{3}{5}$; then the product, 24, is five times the true product, and to get the true product, divide by 5, or indicate it by placing 5 as a denominator.

$$8\times\frac{3}{5}=\frac{8\times 3}{5}\text{ or }\frac{24}{5}.$$

III. To multiply a fraction by a fraction:

$$\frac{3}{7}\times\frac{5}{8}=\frac{3\times 5}{7\times 8}=\frac{15}{56}.$$

$$\frac{3}{7}\times 5=\frac{3\times 5}{7}=\frac{15}{7},\ \frac{15}{7}\div 8=\frac{15}{7\times 8}=\frac{15}{56}.$$

ILLUSTRATED: Multiply $\frac{3}{7}$ by 5 as a whole number, getting a product 8 times as large as it should be, and to get the true product divide by 8, which is done by multiplying the denominator, 7. Observation shows that the first numerator has been multiplied by a number equal to the second, and the first denominator has been multiplied by a number equal to the second; hence place the product of the numerators over the product of the denominators.

IV. To multiply a mixed number by an integer, multiply the fractional part, then the integral, by the multiplier; as, 8 times $\frac{3}{5}$ is $4\frac{4}{5}$, write $\frac{4}{5}$; 8 times 7 are 56, 56 plus 4 equals 60. Continue as with the multiplication of integers.

$$\begin{array}{r} 27\frac{3}{5} \\ 8 \\ \hline 220\frac{4}{5} \end{array} \qquad \begin{array}{r} 856\frac{7}{8} \\ 27 \\ \hline 8)189 \\ \hline 23\frac{5}{8} \\ 5992\phantom{\frac{5}{8}} \\ 1712\phantom{0\frac{5}{8}} \\ \hline 23135\frac{5}{8} \end{array}$$

V. To multiply an integer by a mixed number, multiply the integer by the fractional, then by the integral, part, and add the partial products; as,

$$\begin{array}{r} 456 \\ 23\frac{2}{3} \\ \hline 3)912 \\ \hline 304 \\ 1368 \\ 912 \\ \hline 10792 \end{array}$$

VI. To multiply a mixed number by a mixed number, multiply by the fraction then by the integral parts of the multiplier; as,

Multiply the *md* by 7, and place the product, $3358\frac{1}{4}$, below the line; divide this by 8 to obtain the product by $\frac{7}{8}$; multiply by 87; 87 times 3, or 3 times 87, equals 261, which is placed above the *md* and divided by 4 to obtain the product of $\frac{3}{4}$ by 87, which is placed as one of the partial products.

Multiply the integral parts and find the sum of the partial products.

$$\begin{array}{rl} 4)261 & \\ \hline 479\frac{3}{4} & \\ 87\frac{7}{8} & \\ \hline 8)3358\frac{1}{4} & \\ \hline 419\frac{25}{32} & 25 \\ 65\frac{1}{4} & 8 \\ 3353\phantom{\frac{1}{4}} & \\ 3832\phantom{0\frac{1}{4}} & \\ \hline 42158\frac{1}{32} & (\frac{33}{32} = 1\frac{1}{32}) \end{array}$$

Or make the mixed numbers fractions, and proceed as in Case III.; as, $4\frac{2}{3} \times 5\frac{7}{8} = \frac{14}{3} \times \frac{47}{8}$, etc.

To simplify a compound fraction, consider the word *of* equivalent to the sign, ×, and apply Case III. $\frac{3}{4}$ of $\frac{5}{7} = \frac{3}{4} \times \frac{5}{7} = \frac{15}{28}$.

DIVISION OF FRACTIONS.

I. To divide a fraction by an integer, divide the numerator or multiply the denominator by the integer; as,

$$\frac{6}{7} \div 3 = \frac{6 \div 3}{7} = \frac{2}{7}. \qquad \frac{6}{7} \div 5 = \frac{6}{7 \times 5} = \frac{6}{35}.$$

PRIN. Dividing the *dd* (num.), etc.
Multiplying the *dr* (denom.), etc.

II. To divide an integer by a fraction, divide the integer by the numerator and multiply the quotient by the denominator of the fraction; as,

$$8 \div \frac{3}{5}. \quad 8 \div 3 = \frac{8}{3}, \ \frac{8}{3} \times 5 = \frac{8 \times 5}{3} = \frac{40}{3}, \text{ or } 8 \div \frac{3}{5} = 8 \times \frac{5}{3} = 13\tfrac{1}{3}.$$

Dividing by 3, a number 5 times as large as the true divisor, gives a quotient $\frac{1}{5}$ as large as the true quotient, and to get the true quotient, multiply by 5.

Observation shows that the same result is obtained by inverting the divisor and multiplying.

III. To divide a mixed number by an integer, divide the integral part and then the fractional part by the integer; should there be a remainder of the integral part, make it, with the fractional part, a fraction, and divide.

$$\begin{array}{l} 5\underline{)35\tfrac{2}{3}} \\ \quad 7\tfrac{2}{15} \end{array} \quad \begin{array}{l} 5\underline{)37\tfrac{2}{3}} \\ \quad 7\tfrac{8}{15} \end{array} \left(\begin{array}{r} 2\tfrac{2}{3} \div 5 \\ \tfrac{8}{3} \div 5 \end{array} \right)$$

IV. To divide an integer by a mixed number, multiply both *dd* and *dr* by a number equal to the denominator of the fractional part, and divide as in whole numbers; as,

$$\begin{array}{l} 3\frac{1}{2})475 \\ \frac{2}{7}\quad \underline{\quad 2} \\ \quad\ \ \underline{)950} \\ \quad\ \ 135\frac{5}{7} \end{array} \quad \text{or} \quad \begin{array}{l} 475 \div 3\frac{1}{2} = \\ 475 \div \frac{7}{2} = \\ 475 \times \frac{2}{7} \text{ or } \frac{950}{7}, \text{ or } 135\frac{5}{7}. \end{array}$$

Or, may make the *dr* a fraction and apply Case II. Do not put an integer into the form of a fraction by placing 1 for a denominator. (While developing fractions, do not change fractions to decimals.)

V. To divide a fraction by a fraction, divide the *dd* by a number equal to the numerator of the *dr*, and multiply the *quo* by a number equal to the denominator of the *dr*. Apply principles given.

$$\frac{3}{7} \div \frac{4}{5} = \frac{3}{7} \times \frac{5}{4} = \frac{15}{28}.$$

$$\frac{3}{7} \div 4 = \frac{3}{7 \times 4} = \frac{3}{28}.$$

$$\frac{3}{28} \times 5 = \frac{3 \times 5}{28} = \frac{15}{28}.$$

Observation shows the work to be equivalent to inverting the *dr* and multiplying.

VI. To divide a mixed number by a mixed number, multiply both *dd* and *dr* by the l. c. m. of the denominators of the fractions, and proceed as with whole numbers; or reduce both *dd* and *dr* to fractions and apply Case V.

$$\begin{array}{l} 17\frac{2}{3})\ 87\frac{3}{4} \\ \underline{\ 12\ }\ \ \underline{\ \ 12\ } \\ 212)1053(4\frac{205}{212} \\ \qquad\ \ \underline{848} \\ \qquad\ \ 205 \end{array} \quad \text{or} \quad \begin{array}{l} 87\frac{3}{4} \div 17\frac{2}{3} \\ \underline{\ \ 4\ \ }\ \ \ \underline{\ \ 3\ \ } \\ \frac{351}{4} \div \frac{53}{3} = \frac{351}{4} \times \frac{3}{53} = \frac{1053}{212} \\ \\ \qquad\qquad 212)1053(4\frac{205}{212} \end{array}$$

A complex fraction is simplified by multiplying both terms by the l. c. m. of the denominators of its fractional parts; as,

$$\frac{\frac{3}{4} \times 4}{7 \times 4} = \frac{3}{28}. \quad \frac{2\frac{1}{2} \times 4}{\frac{3}{4} \times 4} = \frac{10}{3}, \text{ or } 3\frac{1}{3}.$$

$$\frac{7\frac{3}{4} \times 12}{8\frac{2}{3} \times 12} = \frac{93}{104}, \text{ or } 7\frac{3}{4} \div 8\frac{2}{3}, \text{ etc.}$$

May divide the numerator by the denominator according to one of the cases of division of fractions.

All operations in fractions are combinations of operations with whole numbers.

RELATION OF NUMBERS.

Only like numbers can be compared.

To find what part one number is of another, divide the number compared by that with which it is compared. The *is* number is always the *dd*, or the number on which the question is made. 6 is what part of 9? 6 is the number compared, and is the *dd*; 9 is the number with which it is compared, and is the *dr*.

Three questions can be made from any question in relation of numbers, and each of these changed into a simple example in multiplication.

6 is what part of 9? 6 is $\frac{2}{3}$ of what number? What number is $\frac{2}{3}$ of 9?

$6 = ? \times 9$. $6 = \frac{2}{3} \times ?$ $? = \frac{2}{3} \times 9$.

$Pd = mr \times md$. The three are resolved into simple questions of multiplication. Apply the formulas.

These questions are similar to those formed upon the terms, cost, price, and quantity. Show it.

This same form occurs with fractions, decimals, compound denominate numbers, and in percentage. The principle for each is the same, the only difference being in the numbers used; as,

$\frac{3}{4}$ is what part of $\frac{5}{6}$? Fractions.

.07 is what part of 2.5? Decimals.

2 yd. 1 ft. 7 in. is what part of 1 rd. 3 yd. 2 ft. 5 in.? Compound denominate numbers.

12 is what per cent. (%) of 36? Percentage.

ANALYSIS IN FRACTIONS.

There is no difficulty if a base unit be established.

Ex. A's money equals $\frac{3}{4}$ B's, B's equals $\frac{2}{3}$ C's; all have $ 91. How much has each?

Take C's as a base. C's equals 1 C's money; B's, $\frac{2}{3}$ C's; A's, $\frac{3}{4}$ of $\frac{2}{3}$, $\frac{1}{2}$, C's. All have the sum of 1, $\frac{2}{3}$, and $\frac{1}{2}$ C's money, $\frac{13}{6}$ C's money, $ 91. $\frac{1}{6}$ C's is $\frac{1}{13}$ of $ 91, or $ 7; $\frac{3}{6}$ C's is 3 times $ 7, $ 21, A's; $\frac{4}{6}$ C's is 4 times $ 7, or $ 28, B's; $\frac{6}{6}$ C's is 6 times $ 7, or $ 42, C's.

Verify: The sum of $ 42, $ 28, and $ 21 is $ 91. $\frac{2}{3}$ of $ 42, C's, is $ 28, B's; $\frac{3}{4}$ of $ 28 is $ 21, A's.

(Much depends upon properly expressing the different steps of an operation.) (Attention, attention, attention; think, think, think; language clear, definite, and accurate.)

Ex. 2. A can do a piece of work in 8 days, B in 10 days, C in 12 days. In how many days can they do it by working together?

A can do it in 8 d., or $\frac{1}{8}$ in 1 d.; B can do it in 10 d., or $\frac{1}{10}$ in 1 d.; C can do it in 12 d., or $\frac{1}{12}$ in 1 d. All can do the sum of $\frac{1}{8}$, $\frac{1}{10}$, and $\frac{1}{12}$, or $\frac{37}{120}$, of it in 1 day. It will take as many days to do $\frac{120}{120}$ as $\frac{37}{120}$ is contained times in $\frac{120}{120}$, or $3\frac{9}{27}$ d.

DECIMALS.

A **decimal** is a number at the right of the decimal point (it is a fractional part of a unit); as, .75.

The decimal may be in the integral form or fractional; as, .02, .00$\frac{1}{3}$, .02$\frac{1}{3}$.

A mixed number may have an integral and a decimal part; as, 47.03$\frac{3}{7}$.

The orders in decimals, as in integers, bear a tenfold relation to one another. It requires two figures to express tens; one, tenths; three, hundreds; two, hundredths; etc.

It requires one figure less to express a decimal than it does to express the corresponding orders of integers.

To write a decimal, ascertain the number of figures required for the corresponding whole number; one less gives the number required for the decimal. Take the number of figures given from the number required, and the result shows the number of ciphers required. Write the decimal point, the ciphers needed, and the number given.

Illustrated: Write 27 millionths. The corresponding whole number, millions (which indicates the first order in the third period, or one order more than 2 full periods of 3 figures each), requires 7 figures. The decimal requires 1 less, or 6 figures. Two figures, 27, are given. 2 from 6 leaves 4 orders to be filled with ciphers. Write the decimal point, 4 ciphers and the number given. .000027. Give much drill.

The familiarity with orders and periods of whole numbers makes decimals easy.

Read decimals the same as whole numbers, giving the name of the right-hand order. .04075 is read, four thousand seventy-five (the same as a whole number) hundred-thousandths. *Ths* is annexed to the integral name to make the decimal; as hundred, hundredths; thousand, thousandths.

To reduce fractions to decimals, annex ciphers to the numerator, divide by the denominator, and point off as many decimal places in the quotient as there are ciphers annexed; as, $\frac{3}{7}$, 7)3.000(.428$\frac{4}{7}$. It is simple division continued to the right of the decimal point into decimals.

It may be explained thus: Multiply the fraction $\frac{3}{7}$ by 1 with as many ciphers annexed as decimal places desired and, to keep the value, divide by a number equal to the multiplier, which do by moving the decimal point as many places to the left as there are ciphers in this divisor; as, $\frac{3}{7} \times 1000$ equals $\frac{3000}{7}$ or 428$\frac{4}{7}$; 428$\frac{4}{7} \div 1000$ equals .428$\frac{4}{7}$.

To reduce a decimal to a fraction, multiply the decimal by 1, with as many ciphers annexed as there are decimal places in the number, and to keep the value, divide by an equal number by expressing it as a denominator. Reduce the result to its simplest form; as, $.428\frac{4}{7} \times 1000$ gives $428\frac{4}{7}$.

$$428\tfrac{4}{7} \div 1000 \text{ gives } \frac{428\frac{4}{7}}{1000}. \qquad \frac{428\frac{4}{7} \times 7 = 3000}{1000 \times 7 = 7000} = \frac{3}{7}.$$

Decimals are added and subtracted the same as whole numbers, observing to have the decimal points in a vertical line.

Multiply decimals as though they were whole numbers and point off as many places in the product as there are in both *md* and *mr*.

To make *md* and *mr* integral, multiply each by 1 with as many ciphers annexed as it has decimal places.

Principle: Multiplying both *md* and *mr* multiplies the true product by the product of the secondary multipliers.

To get the true product, divide the product found by the product of the secondary multipliers, which, in decimals, is 1 with as many ciphers annexed as there are decimal places in both *md* and *mr*. To divide by 1 with ciphers annexed, move the decimal point as many places to the left as there are ciphers. Hence, point off as many decimal places, etc.

$$\begin{array}{rcrcr}
.037 & \times & 1000 & = & 37 \\
\underline{.25} & \times & 100 & = & \underline{25} \\
.00925 & \times & 100000 & = & 925 \\
925 & \div & 100000 & = & .00925
\end{array}$$

Division of decimals. Divide as with whole numbers.

Case I. If the *dd* and *dr* have the same number of decimal places, the quotient is a whole number.

Principle: Multiplying both *dd* and *dr* by equal numbers does not change the quotient.

$$\begin{array}{rl}
.015) & .765\ (51 \\
\underline{1000} & \underline{1000} \\
15) & 765\ (51
\end{array}$$

II. If the *dd* has fewer decimal places than the *dr*, annex ciphers to the *dd* to make the same number of decimal places in it as in the *dr*, and apply Case I.; as, .015) 7.65 or .015) 7.650 (510.

Annexing ciphers to a decimal does not change its value;

to multiply by 1 with ciphers annexed, move the decimal point to the right, not annex ciphers (it may be necessary to fill some vacancies with ciphers).

III. If the *dd* has more decimal places than the *dr*, point off as many in the *quo* as there are more in the *dd* than in the *dr*; as, .15) .765 (5.1; .15 × 100) .765 × 100, 15) 76.5 (5.1; 15) 76.5 × 10, 15) 765 (51, 51 ÷ 10 = 5.1.

Multiplying both *dd* and *dr* by 100 does not change the *quo*. Multiplying the *dd* by 10 multiplies the *quo* by 10, and to get the true *quo*, divide the *quo* found by 10, by moving the decimal point.

CIRCULATING DECIMALS.

A decimal in which the figures regularly repeat is a **circulating decimal**, and the part repeating, a **repetend.** The repetend may be made a fraction by placing it for a numerator, and as many 9's as there are figures in the repetend, for the denominator.

$\frac{1}{7} = .142857142$, etc., or $.\dot{1}4285\dot{7}$; $.\dot{1}4285\dot{7} = \frac{142857}{999999}$, or $\frac{1}{7}$. $.13\dot{8}\dot{1} = .13\frac{81}{99}$, $.\dot{7} = \frac{7}{9}$. The repetend is indicated by a dot placed over its first and last figures, or over the figure, if but one.

Give circulating decimals but little attention.

CONTRACTIONS.

There are, frequently, short methods for performing work with numbers.

An **aliquot part** is an exact divisor. The aliquot parts of 100 and 1000 are often made use of in contracted work. $12\frac{1}{2}$, $16\frac{2}{3}$, 20, 25, $33\frac{1}{3}$, etc., are aliquot parts of 100.

CONTRACTED MULTIPLICATION.

To multiply by $16\frac{2}{3}$, multiply by 100, a number 6 times as large as the true *mr*, $16\frac{2}{3}$, and divide by 6 to get the true product.

In a similar manner multiply by 20, 25, etc. Do this work mentally.

$978 \times 33\frac{1}{3}$; $978 \times 100 = 97800$; $97800 \div 3 = 32600$. Multiply by 100 for convenience, by moving the decimal point two places to the right. As 100 is 3 times the true *mr*, the *pd* is 3 times the true *pd*, and to get the true *pd*, divide by 3. In short, move the decimal point two places to the right and divide by 3.

To multiply by 11, repeat the multiplicand and place it one order to the right or left and add it to the multiplicand; as,

Long form. Short form.

$$\begin{array}{r} 478\\ 11\\ \hline 478\\ 478\\ \hline 5258 \end{array} \qquad \begin{array}{r} \cdot\ 478\\ 478\\ \hline 5258 \end{array}$$

To multiply by a number consisting of two figures, and the right is 1, multiply by the left, and place its product one order to the left and add to the *md*.

This is really multiplying by 30 and adding to 1 times the multiplicand.

784×31.

Common. Short.

$$\begin{array}{r} 784\\ 31\\ \hline 784\\ 2352\\ \hline 24304 \end{array} \qquad \begin{array}{r} 784\\ 2352\\ \hline 24304 \end{array}$$

To multiply by a number consisting of two figures, and the left is 1, multiply by the right and place its product one order to the right and add to the *md*.

Common. Short.

$$784 \times 13. \quad \begin{array}{r} 784\\ 13\\ \hline 2352\\ 784\\ \hline 10192 \end{array} \quad \text{or} \quad \begin{array}{r} 874\\ 13\\ \hline 784\\ 2352\\ \hline 10192 \end{array} \qquad \begin{array}{r} 784\\ 2352\\ \hline 10192 \end{array}$$

This is really multiplying by the unit figure and adding to 1 ten times the multiplicand. Multipliers, 13, 15, etc.

To multiply by a number whose part is a multiple of another part of it; as, 328. 32 is a multiple of 8: multiply by 8, and its product by 4, placing the partial products in their places and add; as, 4875 × 357.

Long form.

$$\begin{array}{r} 4875 \\ 357 \\ \hline 34125 \\ 24375 \\ 14625 \\ \hline 1740375 \end{array}$$

Short form.

$$\begin{array}{r} 4875 \\ 357 \\ \hline 34125 \\ 170625 \\ \hline 1740375 \end{array}$$

To multiply by such numbers as 99, 998, 997, multiply by 1 with as many ciphers annexed as there are figures in the true *mr*, and from this product take as many times the *md* as there are units indicated by the difference between the true *mr* and the new *mr*, as in the illustration. The multiplication by 1000 and 3 should be done mentally, writing only their results and difference of the same.

Long form.

$$\begin{array}{r} 8756 \\ 997 \\ \hline 61292 \\ 78804 \\ 78804 \\ \hline 8729732 \end{array}$$

Short form.

$$\begin{array}{rrcr} 8756 \times & 1000 & = & 8756000 \\ 8756 \times & 3 & = & 26268 \\ \cline{2-2}\cline{4-4} 8756 \times & 997 & = & 8729732 \\ & 997 & = & 1000 - 3 \end{array}$$

To multiply such numbers as $4\frac{1}{2}$ by $4\frac{1}{2}$, $7\frac{1}{2}$ by $7\frac{1}{2}$ (taking a mixed number whose fractional part is $\frac{1}{2}$, twice as a factor), multiply the fractional parts, and one of the integers by the next higher; as, $7\frac{1}{2}$ by $7\frac{1}{2} = 56\frac{1}{4}$. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, $\frac{1}{2}$ times 7 and 7 times $\frac{1}{2}$ equals 1 times 7. 1 times 7 and 7 times 7 equals 8 times 7, or 56.

CONTRACTED DIVISION.

To divide by 25, $16\frac{2}{3}$, $33\frac{1}{3}$, etc. Divide by 100, by moving the decimal point two places to the left, and multiply this result by the quotient of 100 and the true *dr*; as, $4356 \div 33\frac{1}{3}$: $4356 \div 100 = 43.56$; $43.56 \times 3 = 130.68$. 100 is 3 times the true *dr* and gives a *quo* $\frac{1}{3}$ the true *quo*, and to get the true *quo*, multiply by 3. In short, move the decimal point two places to the left and multiply by 3.

It is sometimes convenient to notice that there are 4 times as many 25's as 100's in a number. 25 is contained in 675, 4 times 6, and 3, or 27 times. Have much mental drill in contractions.

WEIGHTS AND MEASURES.

A **denominate number** is a concrete number; as, 2 apples, 2 ft.

A **simple number** has but one name; as, $ 3, 7 yd.

A **compound denominate number** contains several numbers of different names that may be reduced to the same; as, 2 yd. 1 ft. 7 in.

The chief difference between simple and compound numbers is, that in the former, the scale of reduction is regular (10), while in the latter, the scale varies; as, 237, 2 *h* 3 *t* 7 *u*, 200 *u*, 30 *u* and 7 *u*, or 237 units; 2 yd. 1 ft. 7 in., 72 in., 12 in. and 7 in., or 91 inches.

A line indicates distance. Distances are estimated by linear units.

Table of linear measure: 12 in. make 1 ft., etc.

REDUCTION OF COMPOUND DENOMINATE NUMBERS.

2 yd. 1 ft. 7 in.
3
―
7
12
―
91

To reduce to lower terms, denominations, begin with the highest and multiply by the number that one of that denomination makes of the next lower, and add the number of the next lower, if there be any; multiply this result by the number that one of its name makes of the next lower, and so continue as indicated until the desired result is obtained; as, 2 yd. 1 ft. 7 in. Multiply 2 by 3, as there are 3 times as many feet as yards, and add 1 ft., giving 7 ft. Multiply 7 by 12, as there are 12 times as many inches as feet, and add 7 in., giving 91 in.

To reduce a denominate number to higher terms, divide it by the number it takes of that term to make one of the next higher, and divide each successive integral quotient by the number it takes of its term to make one of the next higher. The last quotient and the several remainders, with their proper names, will be the number required; as,

$$\begin{array}{l} 12\underline{)91\text{ in.}}\text{—}7\text{ in.} \\ 3\underline{)7\text{ ft.}}\text{—}1\text{ ft.} \\ 2\text{ yd.} \end{array}$$

Divide 91 by 12, as it takes 12 in. to make 1 ft., giving 7 ft. 7 in. Divide 7 by 3, as it takes 3 ft. to make 1 yd., giving 2 yd. 1 ft. *Ans.* 2 yd. 1 ft. 7 in.

These two cases may be involved in the same question, but the principles remain the same.

To reduce one compound denominate number to the fractional part of another, use just the same principle as used with simple numbers; divide the number compared by the number with which it is compared, first expressing the work in the form of a fraction, then reducing both terms to like units; as, 3 yd. 2 ft. 5 in. is what part of 4 rd. 2 yd. 1 ft.?

$$\frac{3\text{ yd. }2\text{ ft. }5\text{ in.}}{4\text{ rd. }2\text{ yd. }1\text{ ft.}}=\frac{137\text{ in.}}{876\text{ in.}},\ \frac{137}{876}.$$

2 ft. 8 in. is what part of 1 rd.? $\frac{2\text{ ft. }8\text{ in.}}{1\text{ rd.}}$

2 ft. 8 in. = ? × 1 rd.,

To reduce one compound denominate number to the decimal part of another, reduce it to a fractional part, and this to a decimal; as, 3 yd. 2 ft. 5 in. is what decimal part of 4 rd. 2 yd. 1 ft.? By the preceding case, the fractional part is found to be $\frac{137}{876}$, which equals $.156\frac{86}{219}$.

Or, to reduce a compound denominate number to the decimal part of a unit of a higher denomination, the numbers may be arranged in a vertical column, with the lowest denomination first, and apply the principle of reducing to higher terms; as, Reduce 3 yd. 2 ft. 5 in. to the decimal part of a mile.

$$\begin{array}{l} 12\underline{)5.0000}\text{ in.} \\ 3\underline{)2.4166\frac{2}{3}}\text{ ft.} \\ 5\frac{1}{2})3.8055\frac{5}{9}\text{ yd.} \\ \underline{2\qquad\qquad\quad 2} \\ 11\underline{)7.6111\frac{1}{9}} \\ 320)\ .6919\frac{19}{99}\text{ rd., etc.} \end{array}$$

The same principles are true for all the different tables. Fix them.

SURVEYORS' MEASURE.

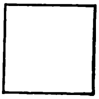
Who are surveyors? What instruments do they use? Compare the table with that of linear measure.

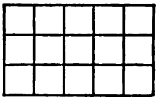
SURFACE MEASURE.

Surface is the outside, and has length and breadth, but not thickness. These measurements are called dimensions.

A **rectangle** is a surface bounded by four straight lines, and has four square corners, right angles; as,

A **square** is a rectangle whose sides are equal; as,





A **surface** is measured by surface, or square, units.

In the figure there are 5 square units in a row, and 3 rows, giving 3 × 5 × 1 sq. unit; or, multiply the number of units in length, linear units, by the number of units in width, to find the number of surface units.

A **square foot** is 12 in. long and 12 in. wide, and contains 144 sq. in.

A **square yard** equals 3 × 3 × 1 sq. ft., or 9 sq. ft.

Compare surface measure with linear to ascertain how its table is formed. Compare surveyors' surface measure with surface measure and with surveyors' long measure.

CARPETING.

Examine a carpeted floor to see how a carpet is laid.

In carpeting a floor, one dimension is filled with breadths of strips, fractions being counted as a whole unit, and the other, with lengths of strips, which are exact. If in making the carpet, the strips would not match when placed even at one end, an allowance is made in the length of each strip excepting the first.

To find the number of yards required to carpet a room: (1) strips placed lengthwise, divide the width of the room, in feet, by the width of the breadth, in feet (a fraction in the quotient being counted as one), to find the number of strips; divide the length by 3 to find the number of yards in one strip (this must be exact and may have a fraction in it). The exact quotient should be multiplied by the number of strips, and to this product add the allowance, if there be any.

If the strips run crosswise, the breadths of strips must fill the length of the room.

Illustrated: Room 19 ft. by 22 ft., carpet $\frac{3}{4}$ yd. wide and stretched lengthwise. $\frac{3}{4}$ yd. equals $\frac{3}{4}$ of 3 ft., or $2\frac{1}{4}$ ft.

$2\frac{1}{4})19(9$, number of strips.
$3)22(\underline{7\frac{1}{3}}$ yds. in a strip.
66 yds. of carpet.

If $\frac{1}{6}$ yd. be allowed for matching, add $8 \times \frac{1}{6}$ to above answer.

PLASTERING.

With a linear measure, foot or yard, find the dimensions of the rectangular surfaces. Find the sum of the surface units in the walls and ceiling; also the surface of the doors, windows, or any other surfaces not to be plastered. Deduct $\frac{1}{2}$ this surface from the entire surface. (As extra time is required to finish around doors, windows, etc., it is customary to allow the mason one-half of this surface.)

The walls may be represented as one long rectangle whose length is the distance around the room and whose width is the height of the room.

Illustrated: Room, 16 ft. long, 14 ft. wide, and 10 ft. high. $(16 + 14) \times 2 = 60$, the number of feet around the room. $10 \times 60 = 600$, or 600 sq. ft., surface of the walls. $16 \times 14 = 224$, or 224 sq. ft., surface of the ceiling; $600 + 224 = 824$, the number of sq. ft. of the entire surface. (At this point, deduct $\frac{1}{2}$ surface of doors, etc.) $824 \div 9 = 91\frac{5}{9}$, or $91\frac{5}{9}$ sq. yd.

A **solid** occupies space and has three dimensions,—length, breadth, and thickness or height.

A **rectangular solid** has six rectangular faces; as, a chalk box.

A **cube** is a rectangular solid with six square faces; as, a die.

A solid is made up of solid, cubic, units. It may be considered as having layers of cubic units, as many as there are linear units in height, each layer having as many rows as there are linear units in width, and each row having as many cubic units as there are linear units in length.

Show it with the blocks and diagrams.

The product of the number of units in each of the three dimensions gives the number of solid units.

ILLUSTRATED: Box 8 in. long, 6 in. wide, and 3 in. deep. There are 8 cubes in a row, 6 rows in a layer, and 3 layers in depth, or 3 × 6 × 8 × 1 cu. in., or 144 cu. in.

Solid measure is used in measuring solids. Compare its table with linear measure. A cubic yard contains 3 × 3 × 3 × 1 cu. ft., or 9 cu. ft.

STONE AND BRICK WORK.

Measurements of walls are made upon the outside, in feet.

Find outside surface, deduct the surface of the openings, and multiply by the thickness, in feet, obtaining the cubic feet of masonry.

To find the number of bricks, multiply the number of cubic feet by 21, as there are 21 times as many bricks as cubic feet.

To find the number of perches, divide the number of cubic feet by 16½, as it takes 16½ cu. ft. to make one perch.

ILLUSTRATED: Building, 40 ft. by 36 ft., and 30 ft. high. $(40+36) \times 2 = 152$, or 152 ft., the full length of the wall. $30 \times 152 = 4560$, the number square feet of outside surface. 1080 sq. ft. represents the surface of doors and windows. $4560 - 1080 = 3480$, the number surface feet of masonry; multiply this by the thickness, and that result by 21, or divide by 16½, as the case may be.

LUMBER.

What is lumber? Its value is how estimated?

Lumber, one inch or less in thickness, is estimated by the surface unit, square foot; for other thicknesses, the number of surface units is taken as many times as there are inches in thickness.

The product of the number of feet in length and width and the number of *inches* in thickness gives the number of board feet. Of tapering boards, find the average width by taking half the sum of the width of the ends, and proceed as before.

Illustrated: Timber, 18 ft. long, 9 in. wide, and 6 in. thick. $18 \times \frac{9}{12} \times 6$, or 81 bd. ft. Board, 16 ft. long, 12 in. wide at one end and 8 in. at the other. $(12 + 8) \div 2 = 10$, the average width. $16 \times \frac{10}{12} = 13\frac{1}{3}$ bd. ft.

LIQUID MEASURE.

What is a liquid?

A unit of liquid measure may be a pint, a quart, a gallon, etc.

Table.

231 cu. in. equals the capacity of one gallon. Dimensions of large vessels may be taken in linear units and the contents found in gallons. Do it.

AVOIRDUPOIS WEIGHT.

The earth has a tendency (tries) to draw all objects to its center, and this force is gravity. The amount of this force on any body is the weight of the body, and its unit of measurement may be a grain, an ounce, or a pound. This measurement is used in weighing ordinary substances; as, grains, spices, animals, etc.

Table. The cental equals the hundredweight.

Custom-house officers allow 2240 lbs. to a ton; this is a long ton.

An avoirdupois pound equals 7000 Troy grains.

TROY WEIGHT.

Troy weight is used in weighing gold, silver, and other precious metals.

Its unit is a grain, a pennyweight, an ounce, or a pound.

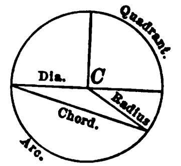
Table.

CIRCULAR MEASURE.

Circular measure deals with the properties of circles.

A circle is a plane bounded by a curved line, every point of which is equally distant from a point called the center.

A radius is the distance from the center to the bounding line, which is the circumference (bearing around). An arc is any part of the circumference. A chord is a straight line joining the ends of an arc. A diameter is one of the longest chords of a circle; it passes through the center. Diameter means "measure through."



For convenience of measurements, every circumference (frequently called circle) is divided into 360 equal parts, or degrees (°); each degree, into 60 parts, or minutes ($'$); each minute, into 60 parts, or seconds ($''$).

One-fourth of a circle is a quadrant.

The earth's turning on its axis carries any point on its surface through a circle. Make the table.

TIME MEASURE.

Time is duration without beginning or ending—known to man.

Nature fixes periods of time which man classifies.

The time required for the earth to turn once on its axis is a day; it turns around a little more than 365 times during one revolution around the sun, which constitutes a year.

Man divides the day into 24 equal parts, hours; the hour into 60 equal parts, minutes; and a minute, into 60 equal parts, seconds.

A **true year** has 365 d. 5 h. 48 m. 49 s. A **common year** has

365 days, making a loss of 5 h. 48 m. 49 s. in 1 yr.; 23 h. 15 m. 16 s., in 4 years. By adding a day to every fourth year, an excess of 44 m. 44 s. is made; and in 100 times that, or 400 years, an excess of a little more than 3 d. would be made: therefore, every fourth year, excepting centennials not divisible by 400, is a leap year of 366 days.

Find different kinds of years, months, and days. Learn the number of days in each calendar month — the little verse in rhyme assists. Make the table.

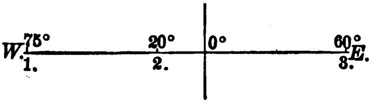
LONGITUDE AND TIME.

Use the globe.

Parallels of latitude extend around the earth parallel to the equator. Meridians are half of meridian circles and extend from pole to pole. Longitude is reckoned from a selected meridian, east and west, on parallels, between meridians.

To find the difference of longitude between two places, if they are both east or west, take the difference of their longitudes; if one is east and the other west, take the sum of their longitudes.

Explain with an erect cross. The vertical line represents the selected meridian; the horizontal, any parallel. Locate each place upon this parallel, and the longitudinal difference is easily found; as,



Dist. 1 to 2 is 75° − 20° = 55°; 2 to 3, 20° + 60° = 80°.

By the rotation of the earth, all places are caused to pass through 360 degrees in 24 hours, or 15 in 1 hour. As the ratio between seconds, minutes, and degrees is the same as that between seconds, minutes, and hours, the ratio between the corresponding terms of longitude and time is 15.

Only difference of longitude can be reduced to a corresponding difference of time, and the reverse. Longitude cannot be reduced to time.

Every unit of time difference represents 15 units space, longitude, difference; and it takes 15 units of space difference to represent 1 unit of time difference.

Much practice in finding difference of longitude and difference of time.

REDUCTION ILLUSTRATED.

$$\begin{array}{rl} 7\text{ h. }35\text{ m. }27\text{ s.} & \text{difference in time of two places.}\\ 15 & \text{ratio number.}\\ \hline 113^\circ\ 51'\ 45'' & \text{difference in longitude.}\end{array}$$

To multiply by 15 and divide by 60 is equivalent to dividing by 4, or taking the number of 4's out of seconds and minutes, for corresponding units of the next higher term.

ILLUSTRATED: $27 = 24 + 3$, $24 \times 15 \div 60 = 6$, $3 \times 15 = 45$; $35 = 32 + 3$, $32 \times 15 \div 60 = 8$, $3 \times 15 = 45$, $45 + 6 = 51$; $7 \times 15 = 105$, $105 + 8 = 113$. Or, there are 6 4's in 27 and 3 remainder, 15 times 3 equals 45; 8 4's in 35 and 3 remainder, 15 times 3 are 45, 45 and 6 are 51; 15 times 7 are 105, 105 and 8 are 113. The names of the terms are obtained by mental analysis.

$$\begin{array}{rl} 15\overline{)113^\circ\ 51'\ 45''} & \text{difference in longitude of two places.}\\ 7\text{ h. }35\text{ m. }27\text{ s.} & \text{difference in time of two places.}\end{array}$$

Multiplying the remainders by 60 and dividing by 15 is equivalent to multiplying by 4, to obtain the corresponding units of the next lower order.

ILLUSTRATED: $113 \div 15 = 7$ and remainder 8, $8 \times 60 \div 15$, or $8 \times 4 = 32$; $51 \div 15 = 3$ and remainder 6, $32 + 3 = 35$, $6 \times 60 \div 15$, or $6 \times 4 = 24$; $45 \div 15 = 3$, $3 + 24 = 27$.

In longitude and time, the multiplication and division should be done mentally. This topic anticipates the four fundamental principles with compound denominate numbers, and should properly follow them, yet it is not so difficult as to cause uneasiness, if left in its present position.

As the sun "rises" in the east, places east of a given place have faster (later) clock time; and places west, slower (earlier) clock time. Add difference of time to the time of any place to find the time of a place east of it; subtract, to find time of a place west of it.

Add difference of longitude to longitude of any place to find the longitude of a place farther east or west; subtract, if less far east or west. Use diagram.

ILLUSTRATED:

Paris, 2° 20′ 22″	E. longitude.	Time?
L.A. 118° 18′	W.	11.30 A.M.
15)120° 38′ 22″	Difference in longitude.	
8 h. 2 m. 33$\frac{7}{15}$ s.	Difference in time.	
11 30	L.A. time.	
7 h. 32 m. 34 s. P.M.	Paris.	

From No. 1 find longitude of L.A.

7 h. 32 m. 33$\frac{7}{15}$ s.	P.M.
11 30	A.M.
8 h. 2 m. 33$\frac{7}{15}$ s.	Difference in time.
15	ratio.
120° 38′ 22″	Difference in longitude.
2° 20′ 22″	E. longitude at Paris.
118° 18′	W. longitude at L.A.

Reference tables. Study them.

Physicians and druggists use Apothecaries' Measure in estimating quantities of medicine. In fluid measure, the unit may be a minim (M) or drop (*gtt*), a dram (ʒ), an ounce (℥), or a pint (O); in dry measure, the unit may be a grain (gr.), a scruple (scr.), a dram (dr.), an ounce (oz.), or a pound (lb.).

THE FUNDAMENTAL PRINCIPLES WITH COMPOUND DENOMINATE NUMBERS.

Addition, subtraction, multiplication, and division of compound denominate numbers differ from similar operations with simple numbers by having an irregular scale of reduction. The general principles are the same.

Illustrated: By addition and multiplication.

h	*t*	*u*
4	7	8
8	4	7
3	8	5
17	1	0

rd.	ft.	in.
4	7	8
8	4	7
3	8	5
16	3½	8
		½=6
16	4	2

h	*t*	*u*
4	2	8
		7
29	9	6

yd.	ft.	in.
4	2	8
		7
34	0	8

5, 12, 20 units. Divide by 10 because it takes 10 of this order to make one of the next higher, etc. 5, 12, 20 inches. Divide by 12 because it takes 12 of this order to make one of the next higher, etc.

7 times 8 *u* are 56 *u*. Divide by 10, as it takes 10 of this order to make one of the next higher, etc.

7 times 8 in. are 56 in. Divide by 12, as it takes 12 of this order to make one of the next higher, etc.

GENERAL ANALYSIS.

In **general analysis**, the number of the same name as the required answer shall be the center around which all work shall cluster.

Arrange the numbers about which all is known, in one column, or line, and those in which the question occurs, in another, — similar terms of the two lines being placed beside each other.

6 men, 8 men, 6 men, 4 days, $ 48.
4 days, 2 days, or 8 men, 2 days, $?
$ 48, $?

No matter how many terms are given, consider only one set of like terms at a time, with the principal term, number like the answer. Work from what is given to unity, then to the number in question line. Take the next set, couplet, in the same manner, and so on until all have been considered. The work should be arranged for cancellation, and performed as indicated.

$$\frac{\overset{8}{\cancel{48}} \times \overset{2}{\cancel{8}} \times 2}{\cancel{6} \times \cancel{4}} = 32, \text{ or } \$\,32.$$

If 6 men earn $ 48, 1 man earns as many dollars, in the same time, as 6 is contained times in 48, — indicate it, — and 8 men earn 8 times as much. If a certain sum — already indicated — is earned in 4 days, in 1 day as many dollars can be earned as 4 is contained times in the given sum, and in 2 days, 2 times as many can be earned. By cancellation, 32 is obtained.

In the analysis, deal with one concrete number the same as with another; with dollars the same as with pounds; as,

$ 450 buy 12 cows,
$ 525 buy ? cows.

$$\frac{12 \times 525}{450} = 14, \text{ or } 14 \text{ cows.}$$

If $ 450 buy 12 cows, $ 1 buys as many as 450 is contained times in 12, indicated, and $ 525 buy 525 times as many. By performing the work as indicated for cancellation, 14 cows is obtained for the answer.

RATIO AND PROPORTION.

Ratio is the relation that one number bears to another of the same kind. (It is similar to Relation of Numbers as considered in connection with fractions, decimals, and compound denominate numbers.) It is indicated by a colon (:) placed between the numbers, — remove the horizontal line in the sign of division and the sign of ratio is left. The sign signifies division; it is the fourth form, previously represented: the first two are generally called division; the third, a fraction; the fourth, a ratio. The terms are, respectively, *dd* and *dr*; numerator and denominator; antecedent and consequent.

Antecedent means "goes before"; consequent, what "follows." The former is the *dd*; the latter, the *dr*. The value of a ratio is found by dividing the first term, antecedent, by the second.

An **indicated ratio** is a couplet (little couple).

A **proportion** is an equality of two or more ratios. The equality is expressed by four dots (a double colon, : :) placed between the ratios. It is equivalent to the sign of equality with the middle of the lines erased; as, $9:3::15:5$ is equivalent to $9 \div 3 = 15 \div 5$, or $\frac{9}{3} = \frac{15}{5}$.

Four terms are required to represent a proportion. They are named in order, first, second, third, and fourth; the first and last are the extremes (outside); the second and third, means (inside): the first and second are the first couplet; the third and fourth, the second couplet: the first and third terms are the antecedents; the second and fourth, the consequents of the couplets.

It is evident that if the fourth term is to be larger than the third, the second must be larger than the first; if smaller, the second must be smaller than the first.

By reducing the fractions $\frac{9}{3}$ and $\frac{15}{5}$ to a common denominator, it will be noticed that the product of the extremes equals the product of the means. This is the general principle of proportion.

It follows that the product of the means divided by one extreme gives the other; or the product of the extremes divided by one mean gives the other.

As only like terms can be compared, in stating a proportion, the third term should be like the required answer. Consider each couplet separately with the third term, and let the question be on the term whose conditions are unknown, and let it be, "Will the answer be more or less than the third term?" if more, the second term of the couplet should be larger than the first; if less, the second should be the smaller. The product of all the means divided by the extreme or the product of all the extremes given, will give the extreme wanted.

The questions of proportion should be arranged as in general analysis, and the work performed by cancellation.

$$\begin{array}{llll} 6 \text{ men}, & 8 \text{ men}, & 6:8::48:? & \frac{48 \times 8 \times 2}{6 \times 4} = 32. \\ 4 \text{ days}, & 2 \text{ days}, & 4:2 & \\ \$48, & \$? & & \end{array}$$

As the answer is to be dollars earned, the third term must be dollars earned, or 48. (Do not ask about 6 men or 4 days, as all is known about them.) Will 8 men earn more or less than 6 men? *Ans.* More. Then the second term must be larger than the first, or 6 : 8. Can more or less be earned in 2 days than in 4 days? *Ans.* Less. Then the second term must be less than the first, or 4 : 2. The product of the means, 8, 2, 48, divided by the product of the extremes given, 6 and 8, gives the other extreme (the fourth term), 32. *Ans.* $ 32.

PARTNERSHIP.

Two or more persons may engage in business, agreeing to share gains and losses in proportion to capital invested or services rendered.

The individuals thus engaged are the partners constituting the firm; and the business is partnership. Capital is the amount invested. The available property is the assets; what the firm owes is the liabilities.

When the assets equal or exceed the liabilities, the firm can pay all its bills and is said to be solvent; otherwise, it is insolvent.

Discuss the business in the class.

In general, the whole capital or number of parts is to each one's capital or number of parts as is the whole gain, loss, or expense to each one's gain, loss, or expense.

$ 400 in business 3 months is equivalent to $ 1200 for 1 month.

Pasturing 25 cows 3 months is equivalent to pasturing 75 cows 1 month.

Capital invested for different periods of time should be changed to equivalents for one unit of time.

Partnership problems may be solved by either proportion or analysis.

ILLUSTRATED: A gains $ 25, B gains $ 35; whole capital is $ 600. What is A's capital? $ 60, whole gain : $ 25, A's gain : :

$600, whole capital : ? ($250, A's capital.) Or A's capital is $\frac{25}{60}$ of $600, or $250.

A invests $250 for 4 months, equivalent to $250 × 4, or $1000 for 1 month.

B invests $400 for 2 months, equivalent to $400 × 2, or $800 for 1 month.

All is equivalent to $1800 for 1 month. They gain $450. What is B's gain? 1800 : 800 :: 450 : ? ($200, B's gain). Or $\frac{800}{1800}$ of $450 is $200, B's gain.

PERCENTAGE.

Percentage means pertaining to the hundred. The terms of percentage are base, rate, and percentage, and are equivalent to *md*, *mr*, and *pd*.

The base is that upon which the percentage is found; the rate, or rate per cent, indicates a number of hundredths of the base; the percentage is the amount found by taking a per cent of the base. % is the sign of per cent and indicates hundredths.

The number of per cent must be reduced to a decimal or a fractional part before multiplying or dividing.

To change any per cent (%) to a decimal part, drop the sign %, which is equivalent to multiplying by 100, and to keep the value, divide by 100, by moving the decimal point two places to the left; as, $7\% \times 100 = 7$, $7 \div 100 = .07$; therefore 7% equals .07, or $\frac{7}{100}$.

To change a decimal part to per cent, multiply by 100 by moving the decimal two places to the right, and to restore the value, affix the sign %, which is equivalent to dividing by 100; as, $.07 \times 100 = 7$, $7 \div 100$ is represented by 7%.

The principles and operations of percentage are like those of multiplication.

$$p = r \times b \qquad r = p \div b \qquad b = p \div r$$

$$pd = mr \times md \qquad mr = pd \div md \qquad md = pd \div mr$$

The general forms are similar to those of relation of numbers.

6 is what part of 12?	6 is what per cent of 12?	6 is $\frac{1}{2}$ of what?
$6 = ? \times 12.$	$6 = ? \times 12.$	$6 = \frac{1}{2} \times ?$
$pd = mr \times md.$	$p = r \times b.$	$pd = mr \times md.$
$mr = pd \div md.$	$r = p \div b.$	$md = pd \div mr.$
$?(\frac{1}{2}) = 6 \div 12.$	$?(50\%) = 6 \div 12.$	$?(12) = 6 \div \frac{1}{2}.$

6 is 50% of what?	What is $\frac{1}{2}$ of 12?	What is 50% of 12?
$6 = .50 \times ?$	$? = \frac{1}{2} \times 12.$	$? = .50 \times 12.$
$p = r \times b.$	$pd = mr \times md.$	$p = r \times b.$
$b = p \div r.$		
$?(12) = 6 \div .50.$		

The three examples illustrate all the principles of percentage, except adding or subtracting to find results above or below the base.

First be sure to understand the business transaction in the different cases under the general head of percentage, then make the question, mentally or otherwise, according to one of the above forms. (Be sure that the first case is mastered, but do not stop on it so long as to give the impression that it is difficult.)

Reduce all succeeding cases to the forms of simple percentage.

PROFIT AND LOSS.

Anything that sells for more than cost gives a gain (*g*), profit; for less, a loss (*l*). The amount of gain is the difference between the cost and the selling price; of loss, the difference between the selling price and cost.

The cost (*c*) is the amount paid by the purchaser.

The selling price (*sp*) is the amount received by the seller.

Discount (*disc*) is the amount taken off from the marked price by the seller. The price marked on goods is the asking or marked price (*mp*).

g is *g*% of *c*, *l* is *l*% of *c*, *sp* is *s*% of *c*, *disc* is *disc*% of *mp*.

The form, p is $r\%$ of b, applies to each of the above statements.

ILLUSTRATED: An article cost \$5 and sold for \$6; find gain per cent.

The difference between \$5, c, and \$6, sp, gives \$1, g. g is $g\%$ of c, \$1 is ?% of \$5, $1 = ? \times 5$; $p = r \times b$, $g = r \times c$, $pd = mr \times md$, $r = g \div c$, $? = 1 \div 5$, or .20, or 20%.

The base, cost in p and l, is represented by 100%. 100% plus $g\%$ or minus $l\%$ equals $s\%$; c plus g or minus l equals sp.

COMMISSION.

For convenience and profit, people often employ some one to buy and sell goods for them. A man in the country may send his products to a man in the city, who will sell them for him; or he may ship a carload of oranges to a man in Chicago, who will sell them for him. The party selling receives a certain per cent of the sales for his pay, commission, and sends the balance, proceeds, to the man from whom he received the oranges. Or one man may send to another to buy goods for him. The man purchasing or selling for another is an agent, or commission merchant, who receives a percentage on selling price or purchase price, as he sells or buys. This is commission business, and always includes three or more parties.

Remittance to consignor, party sending goods to be sold, is the value less commission; remittance to an agent, commission merchant, is the value of the goods bought plus the commission.

The terms are commission, *com*, value of goods, *val*, and rate of *com*, *r*.

The principles and formulas are the same as those of simple percentage.

ILLUSTRATED: *Com* is *com*% of *val*, p is $r\%$ of b, etc.

Val is 100%. *Rem* to agent is 100% plus *com*%. *Rem* to consignor is 100% less *com*%.

An agent sold a quantity of oranges for \$1500, and charged 2% commission. Find commission and remittance.

What is 2% of \$1500?

$?(\$30) = .02 \times \$1500.$

$rem = val - com.$

$?(\$1470) = 1500 - 30.$

Or

Rem% to consignor is 100% minus *com*%.

$?(98\%) = 100\% - 2\%.$

Rem is *rem*% of *val*. What is 98% of \$1500?

$?(\$1470) = .98 \times 1500.$

$com = val - rem.$

$?(\$30) = 1500 - 1470.$

Sent to an agent \$2550 with which to purchase goods after deducting his commission of 2%. Find *com* and *val* of goods.

Rem is *rem*% of *val*.

$val = rem \div rem\%.$

$?(\$2500) = 2550 \div 1.02.$

$com = rem - val.$

$?(\$50) = 2550 - 2500.$

Rem% to agent is 100% plus *com*%.

$?(102\%) = 100\% + 2\%.$

If the property bought and sold be stocks, bonds, etc., the agent is a broker; his allowance, brokerage; and the business, brokerage. Its principles are the same as those of commission.

INSURANCE.

Companies are organized for the purpose of profit to themselves, and to aid people who meet with losses by fire, water, accident, sickness, or death. They are insurance companies and receive names according to their special business. We are to consider only fire insurance.

A stock company has a definite amount of capital represented, and this is divided into a certain number of parts, called shares. Those who buy shares become members, and each has as many votes as he has number of shares, and participates in the gains and losses in proportion to his number of shares.

There are, at least, two parties interested in insurance, the party who insures and the party insured. The amount paid, premium, by the insured parties must be sufficiently large to pay all losses and expenses connected with the business.

It is not presumed that any company should insure for the full value of the property. Why?

In case of total loss of property, the insurance company loses the insured value and expenses less the premium. The insured party loses the value of his property plus the premium less the amount received from the insurance company (insured value).

Usually, the premium is a per cent of the insured value.

The policy is the written agreement, given by the company, and showing the amount of insurance, the premium, term, and condition of the risk.

prem is *prem* % of *ins val*, policy; *p* is *r* % of *b*; *pd* is *mr* times *md*.

QUESTION. *ins val*, \$1500; *r*, $1\frac{2}{3}\%$. Find *prem*.

prem = *r* × *ins val*. ? $= .01\frac{2}{3} \times 1500$, or 25, \$25.

r = *prem* ÷ *ins val*. ? $= 25 \div 1500$, or $.01\frac{2}{3}$, $1\frac{2}{3}\%$.

ins val = *prem* ÷ *r*. ? $= 25 \div .01\frac{2}{3}$, or 1500, \$1500.

TAXES.

To meet public expenses, — name some of them, — all property holders and voters less than 60 yr. old pay a fixed amount of money, tax. Taxable property consists of real estate (immovable) and personal (movable) property. Give examples of each..

A **tax on property** is property tax; on individuals, voters, poll tax; on business, license tax.

The parties who review property and make an estimate of its value are assessors; the amount estimated, an assessment. The parties collecting the amount assessed are collectors; the amount collected, tax.

Collectors are sometimes paid a fixed salary; sometimes a commission.

The commission is estimated as a per cent of the amount collected, in which case the amount used for other expenses is 100% less % paid for collecting, or amount required for given expenses is 100 % less *com* % of amount to be collected. Amount to be collected, tax, is tax % of assessed value.

Illustrated: Assessed value, $4000; rate of tax, 2%.

$\text{Tax} = r \times \textit{as val}.$ $\quad ? = .02 \times 4000$, or 80.
$r = \text{tax} \div \textit{as val}.$ $\quad ? = 80 \div 4000$, or .02, 2%.
$\textit{as val} = \text{tax} \div r.$ $\quad ? = 80 \div .02$, or 4000, $4000.

In some states tax is rated upon $100 instead of $1 as the unit. Tax rate at $1.60 means $1.60 is the tax upon each $100 of assessed value.

It is equivalent to 1.60 ÷ 100, or $$.01\frac{3}{5}$ on every dollar of *as val*, or $1\frac{3}{5}$%.

DUTIES.

Much of the expense of the United States is paid by a tax upon imported goods. It is customs or duties or tariff. If the duty be to pay expenses only, it is a revenue tariff; if to pay expenses and, at the same time, protect home products, it is a protective tariff.

Duties may be assessed upon the value (ad valorem) or upon the articles without regard to their value (specific), or both.

The port at which duties are collected is a port of entry, — name one; the parties that make the collections are customs officers; and their place of business is a custom-house.

Gross weight includes the weight of the goods and boxes, etc.

Tare is an allowance made for weight of boxes, etc.

Net weight is the weight of the goods only.

The long ton of 2240 pounds is used in estimating weight.

Question. Value of goods, $2500; rate of duty, 4%. Find duty.

Duty is duty % of value.

$d = r \times val.$ $? = .04 \times 2500$, or 100, \$100.
$r = d \div val.$ $? = 100 \div 2500$, or .04, 4%.
$val = d \div r.$ $? = 100 \div .04$, or 2500, \$2500.

Specific duty equals rate times number of units.

STOCKS.

Parties desiring to do banking, to build railroads, water systems, etc., form a company according to the laws of the state or nation. A company thus formed is a corporation. The money represented in the company is the stock, or capital. This is divided into parts, or shares, of \$100 each, for nearly all kinds of stocks except mining. The first, or nominal value, is at par. The market value is the selling value. Above par indicates a premium; below par a discount. 117, or at 117, indicates \$117 per share.

Persons owning shares are stockholders.

The excess of expense over profits gives a loss, and suggests an assessment. The excess of profits over expenses gives a dividend.

Dividends and assessments are divided among the stockholders in proportion to the number of shares held by each, not according to the value of the shares; the rate is per the par value.

The agent's compensation for buying and selling stocks is brokerage, and is reckoned on the par value, except for mining stock, in which case it is a percentage of the market value.

A 5% stock pays 5% of \$100, or \$5 on a share. The dividend per share is the income on the cost of a share.

Dividend is what % of cost? \$5 is what % of \$125, cost of one share? $5 \div 125 = .04$, or 4%; thus, in this case a 5% dividend pays 4% on investment, as a \$100 share costs \$125. $dd \div r$ on investment = investment. $5 \div .04 = 125$.

Cost is market value plus brokerage. Market value less brokerage gives net value of sales.

As brokerage is reckoned on the par value, the rate of brokerage always indicates, at a glance, the amount of brokerage per share.

Add the brokerage per share to the market value to find cost value; as,

Stocks at 120, brokerage ¼%, cost $120 plus $¼, or $120¼.

Subtract the brokerage per share from the market value to find net value of sale; as stocks at 120, brokerage ¼%, gives net value $120 less $¼, or $119¾. One may buy and sell stocks the same as he would buy and sell horses and carriages.

ILLUSTRATED: Buy stocks at 112, and pay ¼% brokerage, cost $112¼. Sell the same stocks at 108, and pay ¼% brokerage, net sale price, $107¾. Loss per share, $4½.

Buy a carriage for $112, and pay $¼ to have it delivered, cost $112¼. Sell the carriage for $108, and pay $¼ to have it delivered, net sale price, $107¾. Loss per carriage, $4½.

INTEREST.

One party may lend another a sum of money for which he is to receive a per cent of it for its use. The per cent of the sum loaned depends upon the rate and time.

The sum paid for the use of money is **interest**. The sum borrowed plus the interest makes the **amount** to be paid by the borrower.

The written agreement to pay the sum borrowed, specifying terms and signed by the one who borrows, is a **note**.

$450.75 SAN PEDRO, CAL., March 2d, 1897.

One year after date, for value received, I promise to pay Hollis B. Huntington, or order, four hundred fifty and $\frac{75}{100}$ dollars, with interest at 12 per cent per annum.

S. M. PEDRO.

H. B. Huntington is payee and holder of the note; S. M. Pedro is payer and maker of the note; $450.75, the sum loaned, is the principal; 12 is the rate per cent; one year is the time, but the note may not be paid for several years.

To find the time from the making to the paying of the note, represent the dates as compound denominate numbers, and proceed as in subtraction of the same.

Paid, July 18, 1899.	1899	7	18
Made, Mar. 2, 1897.	1897	3	2
	2 yr.	4 mo.	16 d.

The interest of $1 for the time during which the note remains unpaid indicates the decimal part (per cent) of the principal to obtain the whole interest.

As applied to all cases, the six per cent method is the best. Master this, and then use any shorter method desired in special cases.

Six per cent method. This finds the interest on $1 for the time, at 6%.

Int on $1, for 1 yr., is $.06.
Int on $1, for 1 mo., is $\frac{1}{12}$ of $.06, or $.00½ or $.005.
Int on $1, for 1 d., is $\frac{1}{30}$ of $0.005, or $.000⅙.

(30 days indicate a commercial month; 360 days, 1 year.)

To find the *int* on $1 for the time, at 6%, multiply the *no* of years by .06, the *no* months by .00½, the *no* days by .000⅙, and express their sum as a decimal part of a dollar.

Int on $1, for 2 yr., is 2 × $.06, or	$.12
Int on $1, for 4 mo., is 4 × .00½, or	.02
Int on $1, for 16 d., is 16 × .000⅙, or	.002⅔
	$.142⅔

$$\begin{array}{rl} 2 \times .06 & = .12 \\ 4 \times .00\frac{1}{2} & = .02 \\ 16 \times .000\frac{1}{6} & = \underline{.002\frac{2}{3}} \\ & .142\frac{2}{3} \end{array}$$

Int on $1 for 2 yr. 4 mo. 16 d. is $.142⅔, or .142⅔ (14.2⅔ %) of the principal.

To find *int* on \$1 for any other rate than 6 %, divide the *int* obtained by this method by 6, and multiply the *quo* by the number denoting the rate.

ILLUSTRATED: Taking the previous example:

$6\underline{)\ \$.142\frac{2}{3}}$, *int*, at 6 %, on \$1.
$\$.023\frac{7}{9}$, *int*, at 1 %, on \$1.
8
$\$.190\frac{2}{9}$, *int*, at 8 %, on \$1.

Contracted: for 5 %, find 1 %, and subtract; for 7 %, add 1 % *int*; for 8 %, add $\frac{1}{3}$ of 6 %; for $7\frac{1}{2}$ %, add $\frac{1}{4}$ of 6 %, etc.

$6)\ \$.142\frac{2}{3}$, for 6 %
$.023\frac{7}{9}$, for 1 %
$\$.118\frac{8}{9}$, for 5 %

$6)\ \$.142\frac{2}{3}$, for 6 %
$.023\frac{7}{9}$, for 1 %
$\$.166\frac{4}{9}$, for 7 %

$3)\ \$.142\frac{2}{3}$, for 6 %
$.047\frac{5}{9}$, for 2 %
$\$.190\frac{2}{9}$, for 8 %, etc.

Interest is *int* % (*r* affected by *t*) of *prin*. (*t* is time in years.)

int = (*int* on \$1) *r* × *t*, × *prin*. *prin* = *int* ÷ *int* on \$1.

r = *int* ÷ *int* on *prin* at 1 %. *t* = *int* ÷ *int* on *prin* for 1 yr.

\$1 + *int* of \$1 = *amt* of \$1. *amt* = *prin* + *int*, or *prin* × *amt* of \$1.

prin = *amt* ÷ *amt* of \$1 for the given time and rate.

Present worth corresponds to the principal, and the debt due in some future time is equivalent to the amount.

Present worth (worth now) is such a sum as, put to interest, would amount to the debt of the same time.

pw = debt ÷ *amt* of \$1 for the time and rate.

True *disc* = debt − *pw*, and equals the *int* on the *pw* for the time and rate.

Formulas illustrated:

Time, 2 yr. 6 mo.; *prin*, \$200; *r*, 9 %.

$$\begin{array}{rl}
2 \times .06 &= .12 \\
6 \times .00\tfrac{1}{2} &= \underline{.03} \\
 & 2\underline{).15}, \textit{ int} \text{ on } \$1 \text{ at } 6\,\%. \\
 & \underline{.07\tfrac{1}{2}}, \textit{ int} \text{ on } \$1 \text{ at } 3\,\%. \\
 & .22\tfrac{1}{2}, \textit{ int} \text{ on } \$1 \text{ at } 9\,\%. \\
 & 6).15\;(.02\tfrac{1}{2}, \textit{ int} \text{ at } 1\,\%.
\end{array}$$

int = $.22\frac{1}{2} \times 200$, or \$ 45.
prin = $45 \div .22\frac{1}{2}$, or \$ 200.
$r = 45 \div 5$, or 9, 9 %.
(*int* at 1 %, $.02\frac{1}{2} \times 200$, or 5, \$ 5.)
$t = 45 \div 18$, or $2\frac{1}{2}$, $2\frac{1}{2}$ yr.
(*int* for 1 yr., $.09 \times 200$, \$ 18.)
amt of \$ 1 = \$ 1 + \$ $.22\frac{1}{2}$, or \$ $1.22\frac{1}{2}$.
amt = \$ 200 + \$ 45, or \$ 200 × $1.22\frac{1}{2}$, or \$ 245.
prin = \$ 245 ÷ $1.22\frac{1}{2}$, or \$ 200.
Debt, \$ 245, due in 2 yr. 6 mo.
pw = \$ 245 ÷ \$ $1.22\frac{1}{2}$, \$ 200.
True *disc* = \$ 245 − \$ 200, or \$ 45, which equals the *int* on \$ 200 for the same time and rate.

All government interest is reckoned for exact number of days, and 365 days to the year; this is accurate interest. To find accurate interest, multiply the *prin* by the rate (decimal) times the actual number of days for which the *prin* should draw *int*, and divide the result by 365. Use the principle of cancellation. *acc int* = *prin* × $r \times d \div 365$.

PARTIAL PAYMENTS.

The payer may make payments on a note before it becomes due, or is paid. These payments are acknowledged by the holder's writing the dates of payments and amounts received across the back of the note.

The operation of partial payments is simple interest repeated.

Find the amount due on the note at the time of the first payment (endorsement), subtract the payment, and with the remainder (balance due), as a new *prin*, find the *amt* from this date to the time of the next payment, and so on to time of settlement. Should any payment be less than the *int* due at the time it was made, pass that date, and find the *amt* up to the time of the next payment, and deduct the sum of the payments not deducted at this time. Make notes, and apply principle.

COMPOUND INTEREST.

Compound interest is simple interest added at regular periods, forming a new *prin* each time that an *int* is added; as,

Face of note, $500; time, 2 yr. 7 mo. 24 d.; rate, 6 %; interest compounded annually. Find amount.

Annual: 2 full periods, and 7 mo. 24 d.

	500
	.06
Simple int first period,	30.00
	500
Amt at end first period, or new *prin*,	530
	.06
Simple int second period,	31.80
	530
Amt at end of second period, or new *prin*,	561.80
	.038
	449440
Simple int for remaining time,— less	168540
than full period,	21.3484
	561.80
Amt at end of term,	583.14
Face of note,	500.00
Compound interest,	$83.14

Semi-annually: 5 *hf yr* periods, 1 mo. 24 d.

500, *prin.*
.03
15.00, *simp int* first period.
500
515, *amt* at end first period.
.03
15.45, *simp int* second period.
515
530.45, *amt* at end of second period.
.03
15.9135, *simp int* third period.
530.45
546.3635, *amt* at end of third period.
.03
16.39, *simp int* fourth period.
546.36
562.75, *amt* at end of fourth period.
.03
16.88, *simp int* fifth period.
562.75
579.63, *amt* at end of fifth period.
.008
4.637, *simp int* for remaining time, — less than full period.
579.63
584.27, *amt* at end of term.
500. face of note, or *prin.*
84.27, compound *int* for full time.

BANKING.

Banking is a business carried on by a company for the purpose of receiving deposits of money for safe keeping, and of loaning money for profit. The place of business is a **bank**, and the chief officers are president and cashier.

If the bank be formed according to the U. S. laws, it is a **national** bank and issues national bank bills, which it loans, besides the deposits. If the bank receive deposits of small or great earnings and pay an interest on the same, it is a **savings** bank.

Some banks will discount commercial (business) paper, usually deducting a certain rate of interest for the actual time it holds the paper, and this deduction is made upon the amount that the paper is worth at the date of payment, maturity. An interest-bearing paper is worth its face plus its interest for the time it remains unpaid. A non-interest-bearing paper is worth its face at the date of its expiration.

The amount deducted is **bank discount**; the amount paid by the bank is **bank proceeds**, or **cash value**.

To find bank proceeds: (*a*) On non-interest-bearing paper, deduct simple interest, using the exact number of days the bank holds the paper, from its face. (*b*) On interest-bearing paper, find amount of the note (simple interest method), and from this amount, deduct the simple interest on it for the actual number of days the bank holds it.

ILLUSTRATED: (*A*) Note, $1000; made Jan. 26, 1897; time, 6 months; disc. Mar. 8, 1897; money worth 10%. Note bears 8% interest. Find proceeds. Due July 26, 1897.

Mar. 23		$6\underline{)\,140}$	Face,	1000
Apr. 30	$140 \times .000\frac{1}{6}$	$= .023\frac{1}{3}$		$\underline{.04}$
May 31		$6\underline{).023\frac{1}{3}}$	*int* 6 mo.	$\underline{40.00}$
Jun. 30		$.003\frac{8}{9}$	*amt* note,	$1040
Jul. 26		$\underline{10}$		

140 days, term of discount, $\$.038\frac{8}{9}$, *simp int* on $1.

$$\begin{array}{rl} 1040, & \textit{amt} \text{ bank receives.} \\ \underline{.038\frac{8}{9}} & \\ 9\underline{)8320} & \\ 924 & \\ 8320 & \\ \underline{3120} & \\ 40.44, & \text{bank discount for 140 d.} \end{array}$$

$1040 – 40.44 = $999.56, bank proceeds, cash value.

(*B*) Same question as (*A*), excepting that the note does not bear interest.

$$\begin{array}{rl} \$1000, & \textit{amt} \text{ due July 26, 1897.} \\ \underline{.038\frac{8}{9}} & \\ 9\underline{)8000} & \\ 888 & \\ \underline{38000} & \\ 38.89, & \text{discount.} \end{array}$$

(Merely simple interest repeated on different amounts and using exact number of days in one case.)

$1000 – 38.89 = $961.11, proceeds, cash value.

Depositors draw their money from the bank only by means of orders, — checks, — made payable to themselves or others.

COMMERCIAL DISCOUNT.

Business men who sell large quantities of goods often make a reduction from the marked price. This is made by a per cent off, and sometimes, for certain favorable conditions, several discounts may be made, each succeeding one from a less amount than the preceding.

The process is that of simple percentage.

ILLUSTRATED: Bill of goods, $2500 with 15, 10, and 5 off, for cash. This means a discount of 15 %, and, on the proceeds, a discount of 10 %, and on the proceeds of this another discount of 5 %.

```
   $2500, bill.
      .15
     -----
     375, first discount.
    2500
    -----
    2125, first proceeds.
      .10
   -------
  212.50, second discount.
  2125
  --------
 1912.50, second proceeds.
     .05
   -------
   95.625, third discount.
 1912.50
 ---------
$1816.875, cash value of bill.


   $2500, bill.
      .85, proceeds of $1; 1.00 – .15 = .85.
   ------
   12500
  20000
  -------
 2125.00, first proceeds.
     .90, proceeds of $1; 1.00 – .10 = .90.
 --------
 1912.50, second proceeds.
     .95, proceeds of $1  1.00 – .05 = .95.
   ------
   956250
 1721250
--------
$1816.87, third proceeds, cash value.
```

EXCHANGE.

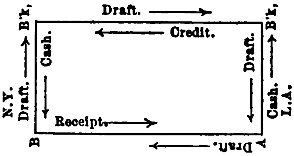
It is often convenient, when one party owes another and a third party owes the first, for the first to request the third to pay the second.

This is done by the first party's giving the second an order for his payment, on the third. The same principle is illustrated by bank checks. The checks are orders on a bank for the payment of another's obligations.

In this form of exchange, payments are made on demand and for face value.

A party in one locality may desire to pay a debt in a distant locality, without sending the money, which might be lost. If a third party in the latter place owes him, he may send an order on said third party, to the second, for payment; otherwise, the first party must call to his aid a party, near his place of business, who has credit in the distant place. This aid will probably be a bank that does an exchange business. He pays the bank for a draft, an order, on the distant bank in favor of his creditor, and forwards it to him. The creditor presents it to his bank and receives payment. The distant bank charges the draft to the bank giving it, and in time, their accounts are settled through the bankers' exchange.

There are four parties interested; the buyer, the maker, the payer, and the payee, — the one to whom paid.



A draft made payable on demand is a **sight draft**; one made payable a certain number of days after demand is a **time draft.**

The time draft always gives a time discount to the one that buys it.

There may be a rate of exchange between the two places, which the maker charges, if it be a premium; or deducts, if it be a discount.

(A person can usually get a draft from the bank at which he does business, without paying any rate of exchange.)

The payee of a time draft presents it to the payer who accepts it by writing across the face of it the word *accepted,* date of same, and his name. The time draft is payable in the number of days specified after it is accepted. Make a draft and have it accepted. It requires four parties. A postal order is a sight draft. Show it. See it.

EXCHANGE ILLUSTRATED:

A, in L. A., wants to pay B, in N. Y., $500. A goes to a L. A. bank and buys a 30 days' draft in favor of B, on a N. Y. bank. The L. A. bank is the maker; the N. Y. bank is the payer; A, the buyer; B, the payee. A sends the draft to B; B presents it to the N. Y. banker, who accepts it. 30 days after it is accepted, B may collect it from the N. Y. bank. Who pays the N. Y. bank? Who gets A's money?

The L. A. bank has the use of A's money 30 days, really longer, before it has to account for it, and should allow A for the use of it, simple interest, at the business rate, which in this case is 9%. Exchange, 1% *prem.* Who pays it?

$1.00
.00¾, *int* on $1 for 30 days, time discount.
$ .99¼, cash cost of $1 draft, were exchange at par.
.01, premium on $1.

$1.00¼, cash cost of $1 *exch*, or 1.00¼ times as many dollars cash as dollars draft. $500 × 1.00¼ = $501.25, cost of draft.

Cost = ($1 − time *disc* + *prem*, or − *disc*) × face.

Face = cost (cash) ÷ cost of $1, as indicated in the parenthesis above. ? = $501.25 ÷ 1.00¼, or 500, $500.

Cost of $1 = cost ÷ face. ? = $501.25 ÷ 500, or 1.00¼, $1.00¼.

Cost of $1 − *prem*, or + *disc* = $1 as affected by time discount.

$1.00¼ − .01 = $.99¼; $1 − .99¼ = .00¾, time discount on $1.

Cost of $1 + time *disc* of $1 = $1 as affected by rate of exchange.

$1.00¼ + .00¾ = $1.01, $1.01 − $1 = $.01, *prem* on $1, or 1%.

AVERAGE OF PAYMENTS.

When bills become due at different times, it may be desirable to pay all of them at one time, and at such a time that will make no loss to either party. Some would be paid after they became due, and others before.

For every amount there is an equivalent unit for a certain time; as, $100 for 4 months is equivalent to $1 for 400 months.

The summary of the times for the different units divided by the total debt gives the equitable time from the earlier date, or from the date of the credit, in which the total debt may be paid.

Illustrated: Credit given March 3, 1897.

$ 400 on 2 months' time equals the use of $1 for 800 months.
$ 300 on 3 do. do. $1 for 900 months.
$ 450 on 5 do. do. $1 for 2250 months.
$1150, all equals the use of $1 for 3950 months,

or $1150 for 3950 ÷ 1150, or 3 mo. 13 d. This time forward from March 3 gives June 16, 1897.

Credit given at different times:

July 5	$300		
Sept. 1	250	$250, 58 days from July 5 is equivalent to $1,	14500 d.
Oct. 15	450	$450, 102 d. from July 5 is equivalent to $1,	45900 d.
	$1000	Total credit is equivalent to $1.00 for	60400 d.

from July 5, or $1000, 60400÷1000, 60 days, or Sept. 3.

AVERAGE.

Summary of cost divided by summary of quantity gives average cost.

In case the average be given, — the cost of the mixture, — take enough of the quantity above and below the mixture price to make gains and losses equal. Unless the quantity be limited by some condition in the question, the answer may be variable.

ILLUSTRATED: Bo't tea; 4# at 50, 2# at 75, and 3# at 60. Find average price: 4# at 50 cost 200; 2# at 75 cost 150; 3# at 60 cost 180. 530, summary of cost, ÷ 9, summary of quantity, gives 58$\frac{8}{9}$, average cost.

$$55\left\{\begin{array}{lllrr} 75 & 1\# & -20 & & 20 \\ 60 & 1 & -\ 5 & & 5 \\ 50 & 1 & +\ 5 & 2\# & 10 \\ 40 & 1 & +15 & & 15 \end{array}\right\}$$

In a mixture of teas costing, respectively, 75, 60, 50, and 40, how many pounds of each may be used to make an average price of 55? (− indicates loss, + indicates gain.) It may take 1# each of 75, 60, and 40, and 2# of 50.

POWERS AND ROOTS.

The terms used are root, exponent, radical sign, index, and power.

The **root** is the number which taken a number of times as a factor gives the power; it is one of the equal factors of a number.

The **exponent** is a small number placed a little to the right and above a number to show how many times the number is taken as a factor.

The **power** is the product obtained by taking the root a number of times as a factor.

The **radical sign** is a character like the letter *r*, $\sqrt{}$, placed at the left and above a number to show that a root is to be taken. The radical sign, alone, indicates the square root.

An **index** is a small figure placed in the radical sign to show what root is to be found.

ILLUSTRATED: $5^3 = 5 \times 5 \times 5$, or 125. 5 is the root;. 3, the exponent; 125, the power.

$\sqrt[3]{125} = 5$. 125 is the power; $\sqrt{\ }$, the radical sign; 3, the index; 5, the root.

The numerator of a fractional exponent indicates a power; the denominator, a root; as, $8^{\frac{2}{3}}$ shows that the 3d root of the 2d power of 8 is to be found, or $\sqrt[3]{8^2}$, or $\sqrt[3]{64} = 4$. $9^{\frac{1}{2}} = \sqrt{9}$.

To find the power of any number, take it as many times as a factor as there are units in the exponent. (Never say, multiply the number by itself, etc., excepting to find the square.) $3^4 = 3 \times 3 \times 3 \times 3$, or 81; $.04^3 = .04 \times .04 \times .04$, or .00064; $(\frac{5}{6})^3 = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$, or $\frac{125}{216}$.

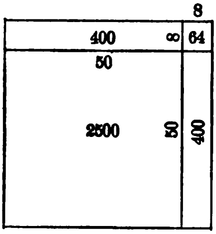
Square root is one of two equal factors of a number. Much mental work with small numbers.

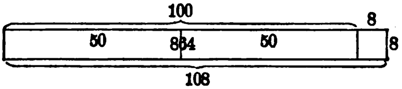
$2^2 = 4$, $9^2 = 81$, $10^2 = 100$, $90^2 = 8100$. Compare the number of figures in the power with the number in the root.

The power contains two times as many figures or two times as many less one as the root; therefore, in extracting the square root, divide the power into periods of two figures each, beginning at the decimal point. Find the greatest square in the left-hand period, and place its root as the first figure in the root. Subtract this square and to the remainder annex the next period, forming a dividend, which divide by two times the root already found, regarded as tens of the next order, place the quotient figure in the root; also, add it to the trial divisor, obtaining the complete divisor. Multiply the complete divisor by the last figure in the root, subtract the product from the dividend used, and to the remainder annex the next period, if there be any, and proceed as before.

```
   3364(58
   25
   ----
100)864
    8
  ---
108)864
    ---
```

ILLUSTRATED: As the power has twice as many figures, or twice as many less one, as the root, the number is divided into periods of two figures each. The greatest square in the first period is 25. Its root is 5, which is placed as the first figure of the root. 25 is taken from the first period, and to the remainder the next period is annexed, giving 864 as the number of surface units in the additions to the previous square. A square may be enlarged by making additions to two adjacent sides, and still be a square. The additions are two rectangles as long as the side of the square and a square corner whose length is equal to the width of the rectangular additions. 5 regarded as tens of the next order, gives 50 as the side of the square, and the length of each rectangular addition, and the length of both of them is 2 × 50, or 100. This is the approximate length of all the additions. Dividing the surface, 864, by 100 gives 8, the width of the rectangles, also the length of the corner. 8 added to 100 gives 108 as the entire length of all the additions, which multiplied by 8, the width, gives 864, the surface of the additions. 58 is one of the two equal factors of 3364.





Use blocks and diagrams to illustrate the work.

Application of square root.

The area of a square is the product of the number of units in length by the number of units in width; the product of two equal numbers or factors.

The area of a square being given to find the length of a side, extract the square root, find one of two equal factors.

CUBE ROOT.

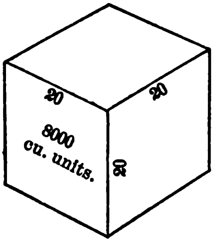
The product of three equal factors is a cube of one of the factors; as, $4^3 = 4 \times 4 \times 4$, or 64.

The **cube root** of any number is one of its three equal factors; as, $\sqrt[3]{64} = 4$. Much mental drill.

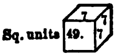
2^3 is 8; 3^3 is 27; 9^3 is 729; 20^3 is 8000; 30^3 is 27,000; 90^3 is 729,000.

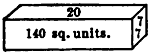
Observation shows that the power contains 3 times as many figures as the root, or 3 times as many less one or two figures; therefore, divide the power into periods of 3 figures each, beginning at the decimal point.

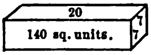
The number of periods indicates the number of figures in the root; as, $\sqrt[3]{19683}$ is 27.

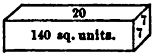


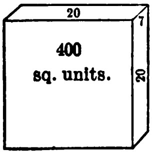
$$\begin{array}{rrrl}
\text{Linear Units.} & \text{Surface.} & \text{Solid.} & \text{Linear Root.} \\
 & & 19683 & (27 \\
 & 20^3 = & \underline{8000} & \\
3 \times 20^2 = & 1200\,) & 11683 & \\
3 \times 7 \times 20 = & 420 & 11683 & \\
7^2 = & \underline{49} & & \\
 & 1669 & & \\
\end{array}$$

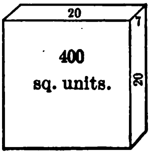


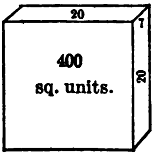












8, under the first period, represents the contents, 8000, of the first cube; 11,683 represents the contents of the 7 additions. 20^2 gives one surface of a large addition, which is a square as long and as wide as the dimensions of the previous cube. 1200 is the surface of the 3 large additions, or the approximate surface of all the additions. 11,683, the number of contents units, divided by 1200, the number of surface units, gives 7, the thickness. Each of the remaining four additions is as wide as the large ones are thick, and three of these are as long as are the large ones, and the 4th is as long as it is wide. The surface of the three is 420, and 49 is one surface of the cubical corner. 1669 represents the outside surface of all the additions, which multiplied by 7 gives their contents, 11,683. 27 is one of the three equal factors, cube root of 19,683. Were there another period, consider the dimensions of the enlarged cube to be 270 and proceed as before.

Therefore, to find the cube root of a number, point it off into periods of three figures each, always beginning at the decimal point. Find the greatest cube in the first period and place its root as the first figure in the root. Take three times the square of the root figure regarded as tens of the next order in the root, for a trial divisor. Place the quotient obtained by this divisor, as the next figure of the root. Add to the trial divisor, three times the last figure multiplied by the number represented by the preceding part of the root, and the square of the last figure of the root, obtaining the complete divisor, which multiply by the last root figure. Should there be more than two periods, consider the root found as tens of the next order to be found and continue as before.

Application.

A cube has three equal dimensions and six equal square faces.

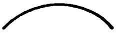
If the contents of a cube is the product of three equal factors, the dimension is found by taking the cube root of the contents.

MENSURATION.

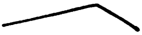
Mensuration pertains to measurements and deals with points, lines, angles, surfaces, and solids.

A point has only position and is represented by a dot (.).

The distance between two points is a line; the shortest, a straight line; as, ________ A line constantly changing its direction is a curved line; as,

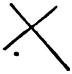


A line abruptly changing its direction is a broken line; as,



A line lying on the surface of still water, or parallel to the surface of still water or the plane of the horizon, is a horizontal line; it is represented by a line extending from left to right.

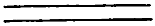
A vertical line is a line pointing towards the center of the earth. It forms a right angle with a horizontal line. It is represented by a line extending towards the top or bottom of the surface on which it is made. A perpendicular line makes a right angle with another line and may have any direction; as,

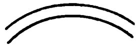


Each line is perpendicular to the other.

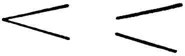
A slanting line (oblique) has any other position than vertical and horizontal.

Parallel lines have the same direction and never could meet, if extended; as,





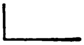
An angle is the difference in direction between two lines or surfaces; as,



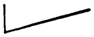
The lines form the sides or legs of the angle, but the length of the lines does not affect the size of the angle. The point where the lines meet, or would meet if extended, is the vertex. Illustrate.

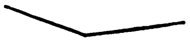
A degree, $\frac{1}{360}$ of a circle, is the unit of angular measurements; the size of the circle does not affect the size of the unit.

An angle formed by perpendiculars and measured by $\frac{1}{4}$ of a circle, 90°, is a right angle; as,



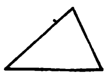
An angle measured by less than 90° is an acute angle; one measured by more is an obtuse angle; as,



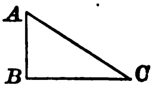


Polygon (*poly*, many; *gon*, angle) is a plane figure having many angles, consequently many sides.

A **triangle** (*tris*, three) is a polygon having three angles and three sides; as,



If the triangle have a right angle, it is a right-angled triangle; as,



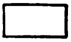
The right angle is at *B*. Either leg of the right angle is the base, and the other the altitude or perpendicular. The side opposite the right angle is the hypothenuse or hypotenuse.

Of other triangles, the sides upon which they rest are usually called their bases, but any side may be taken for the base.

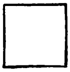
The perpendicular distance from the base to the vertex opposite is the altitude of the triangle. The base, in this connection, is called the width.

A **quadrilateral** (*quattuor*, four; *latus*, sides) is a polygon having four sides.

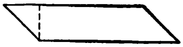
A **rectangle** is a quadrilateral having four right angles; as,



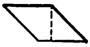
A **square** is a rectangle having four equal sides; as,



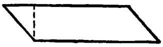
A **parallelogram** is a quadrilateral whose opposite sides are equal and parallel; as,



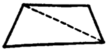
A **rhombus** is a parallelogram, all of whose sides are equal, but having no right angles; as,



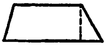
A **rhomboid** is a parallelogram whose opposite sides only are equal, and which has no right angles; as,



A **trapezoid** is a quadrilateral having only two sides parallel; as,



A **trapezium** is a quadrilateral having no two sides parallel; as,

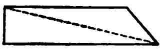


The altitude of a parallelogram is the perpendicular distance from the base to the side opposite; the width is the length of the base.

The altitude of a trapezoid is the distance between its parallel sides; its width is half the sum of the parallel sides.

The trapezium has neither altitude nor length. It must be divided into two triangles before measured.

A **diagonal** (*dia*, through; *gon*, angle) is the distance between any two angles not adjacent (joined to); as,

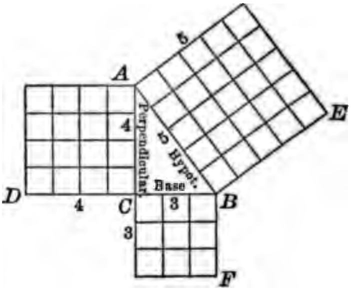


A polygon of five sides is a pentagon; of six sides, a hexagon; of seven, a heptagon; of eight, an octagon; of an infinite number of sides, a circle.

The **perimeter** (*peri*, around; *meter*, measure) is the distance around a polygon. The radius of a polygon is the distance from the center to a vertex; the apothem is the perpendicular distance from the center to a side.

Whenever the word *polygon*, unqualified, is used, a regular polygon is meant. A regular polygon has its sides and angles equal to one another; it is both equilateral and equiangular.

A **circle**, a many-sided polygon, is a plane bounded by a curved line, every point of which is equally distant from a point within, the center. The bounding line is the circumference, and any part of it, an arc. The distance from the center to the circumference is a radius (rays); it is one-half a diameter, the longest straight line in a circle.



A chord is a straight line between the ends of an arc.

Observe that on the sides of a right triangle, *ABC*, squares are erected, and each of these squares is subdivided into equal smaller squares. The number of squares on the hypotenuse, 25, is equal to the sum of the number on the base and perpendicular, 9 and 16; hence the formulas:

$$H^2 = B^2 + P^2, \text{ or } H = \sqrt{B^2 + P^2}, \quad H = \sqrt{3^2 + 4^2}, \text{ or } 5.$$
$$B^2 = H^2 - P^2, \text{ or } B = \sqrt{H^2 - P^2}, \quad B = \sqrt{5^2 - 4^2}, \text{ or } 3.$$
$$P^2 = H^2 - B^2, \text{ or } P = \sqrt{H^2 - B^2}, \quad P = \sqrt{5^2 - 3^2}, \text{ or } 4.$$

Stated: Of right triangles, the hypotenuse equals the square root of the sum of the squares of the other sides; the base equals the square root of the difference of the squares of the other sides; and the perpendicular equals the square root of the difference of the squares of the other sides.

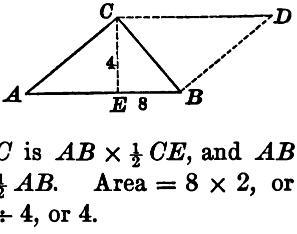
AREAS OF SURFACES.

A rectangle is composed of surface, square, units. There are as many in length as there are linear units in length, and as many rows of these square units as there are linear units in width. Hence, to find the area of a rectangle, take the product of the number of linear units in width and length. Illustrated on page 32.

Any parallelogram is equivalent to a rectangle having an equal base and altitude. To find the area of a parallelogram, take the product of the base (length) and altitude (width). The same is true of a trapezoid, excepting that half the sum of the parallel sides makes the average length which should be multiplied by the altitude.

A triangle is equivalent to one-half of a parallelogram having an equivalent base and altitude.

Multiply the base by one-half the altitude to find the area of a triangle. Triangle *ABC* is one-half of parallelogram *ABDC*. *CE* is the altitude of each. Area of *ABC* is $AB \times \frac{1}{2} CE$, and $AB = \text{area} \div \frac{1}{2} CE$, and $CE = \text{area} \div \frac{1}{2} AB$. Area $= 8 \times 2$, or 16. $AB = 16 \div 2$, or 8. $CE = 16 \div 4$, or 4.



The area of a trapezium is found by dividing it into two triangles; then find the sum of the areas of the two triangles.

By means of its radii, a polygon may be divided into as many triangles as it has sides. The base of each triangle is a side of the polygon; its altitude, the apothem of the polygon. To find the area of a polygon, find the area of its triangles and take their sum. As one factor, the apothem, is constant, it is better to take the sum of the other factors and multiply by the apothem, or multiply the perimeter by one-half the apothem.

Since a circle is a polygon of many sides, and the radius equals the apothem, to find the area of a circle, multiply the circumference by $\frac{1}{2}$ the radius.

(Compare the distance around a circle with its distance through, using a tape measure.)

The circumference of a circle is 3.1416, $3\frac{1}{7}$, times the diameter, $\pi \times d$.

Area of $\mathit{cir} = \mathit{circum} \times \frac{1}{4} \times \mathit{dia}$. Since the $\mathit{circum} = \mathit{dia} \times 3.1416$, area of $\mathit{cir} = \mathit{dia} \times 3.1416 \times \frac{1}{4} \times \mathit{dia}$, or $\mathit{dia}^2 \times .7854$ $(\frac{1}{4} \times 3.1416)$. Or, area of $\mathit{cir} = 2 \times r\ (\mathit{dia}) \times 3.1416 \times \frac{1}{4} \times 2 \times r$, $2 \times 2 \times \frac{1}{4} \times 3.1416 \times r^2$, $\pi \times r^2$.

Area of $\mathit{cir} \div 3.1416(\frac{22}{7}) = r^2$. From r^2 may be found *dia* and *circum.*

SOLIDS.

Draw all the figures on pages 252, 253 of the State Arithmetic to illustrate the following work:

A **prism** is a solid whose convex faces are rectangles and whose bases are two equal, parallel polygons.

A **pyramid** is a solid whose convex faces are triangles, all of which meet at a point, the apex, and whose base is a polygon.

Prisms and pyramids are named by the forms of their bases; a square base, a square prism, pyramid, etc.

A **cone** has a circular base and one convex surface which tapers to a point, the apex.

A **cylinder** has two equal, parallel, circular bases, and one curved, convex surface.

A **frustum** of a cone has two unequal, parallel, circular bases and one curved, convex surface. It may be made by cutting off the top of a cone with a plane parallel to the base.

A frustum of a pyramid has two similar, unequal parallel polygons for bases, and whose convex surfaces are **trapezoids.** It may be made from a pyramid in the same way that a frustum of a cone is made from a cone.

A **sphere** is a solid bounded by a uniformly curved surface, every point of which is equally distant from a point within, the center.

The **altitude** of prisms and frustums is the perpendicular distance between their bases.

The altitude of a pyramid or cone is the perpendicular distance from the base to the apex; the slant height is the perpendicular distance from the apex to the perimeter of the base.

The convex surfaces of prisms and cylinders, peeled off, would be rectangular surfaces, whose lengths are the perimeters and whose widths are the altitudes of the prisms or cylinders. Hence, to find the convex surface of either, take the product of the perimeter and the altitude; to this, add the area of the bases, to find the entire surface. Illustrate.

The convex surface of a pyramid is equivalent to a part of a polygon; that of a cone, to a part of a circle. Hence, to find the convex surface of either, take the product of the perimeter of the base and half the slant height; to find the entire surface, add the surface of the base.

CONE: Slant height, 24; diameter of base, 10.

Convex surface $= 10 \times 3.1416 \times \frac{1}{2}$ of 24, or 376.992.

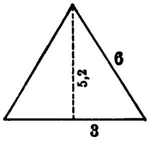
Area base $= 5^2 \times 3.1416$, or 78.54. Entire surface, 455.532.

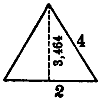
The convex surface of a frustum represents a series of trapezoids.

Show it with paper and drawings.

To find the convex surface of a frustum, take the product of half the sum of its perimeters and the slant height; to this add the sum of the areas of the bases, to find the entire surface.

FRUSTUM OF A HEXAGON: Slant height, 20; lengths of the sides of the bases, respectively, 4 and 6; perimeters, 6×4 and 6×6, 24 and 36; apothems,





$$\sqrt{4^2 - 2^2}, \text{ or } 3.464, \text{ and } \sqrt{6^2 - 3^2}, \text{ or } 5.2.$$

Convex surface $= (24 + 36) \div 2 \times 20 = 600$.

Area upper base $= 24 \times \frac{1}{2} \times 3.464 = 41.568$.

One each of the 6 equal △ of each polygon.

Area lower base $= 36 \times \frac{1}{2} \times 5.2 = 93.6$.

Entire surface $= 735.168$.

Area of a sphere is $4 \times r^2 \times 3.1416$, $4\pi r^2$. (The reason for this and for the processes by which the contents of pyramids, cones, their frustums, and spheres are obtained, will be found in higher geometry.)

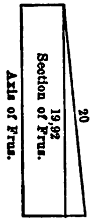
A rectangular solid is composed of cubic units, and the process by which the number is found is explained on page 34.

By the same line of reasoning, the contents of prisms and cylinders are found by multiplying the area of the base by the altitude. It is the same as finding the contents of a wall.

A pyramid or cone is equivalent to one-third of a prism or cylinder having an equal base and altitude. The contents is equal to the area of the base multiplied by one-third the altitude.

The frustum of a pyramid or cone is equivalent to three pyramids or cones whose bases are respectively equal to the upper and lower bases of the frustum and a mean proportional between them, and whose altitude is the same as that of the frustum. To find the contents of a frustum, find the sum of the areas of the two bases and the square root of the product of these areas, and multiply this sum by one-third the altitude of the frustum.

Illustrate, using previous question.



Upper base, 3.464.

Base of *rt* $\triangle$, $5.2 - 3.464 = 1.736$.

Hyp of *rt* $\triangle$, 20.

Alt of *rt* $\triangle$, 19.92.

Lower base, 5.2.

Section made through the lines of the apothems.

Altitude $= \sqrt{20^2 - (5.2 - 3.464)^2} = 19.92$.

Area upper base $= 24 \times \frac{1}{2} \times 3.464 = 41.568$

Area lower base $= 36 \times \frac{1}{2} \times 5.2 = 93.6$

Area mean base $= \sqrt{41.568 \times 93.6} = 62.38$

Contents $= 41.568 + 93.6 + 62.38 \quad (197.548)$

$\times \frac{1}{3} \times 19.92 = 1311.718$.

A sphere is equivalent to an infinite number of cones with their apices at the center, and the sum of whose bases form the surface of the sphere. To find contents of a sphere, multiply the surface by $\frac{1}{3}$ the radius, or $\frac{1}{6}$ the diameter of the sphere.

Cont of sphere $=$ *surf* $\times \frac{1}{3} r$, or $\frac{1}{6} d$. As *surf* $=$ *gt cir* $\times 4$, *cont* of *s* $=$ area of *gt cir* $\times (4 \times \frac{1}{6}) \frac{2}{3} d$.

Area *gt cir* $= 3.1416(\frac{22}{7}) \times d, \ \times \frac{1}{4} d$, or $.7854(\frac{11}{14}) \times d^2$, or $\frac{1}{4} \times \pi d^2$.

Surf sph $= d^2 \times .7854(\frac{11}{14}) \times 4$, or $d^2 \times 3.1416(\frac{22}{7})$, or πd^2.

Cont sph $= d^2 \times 3.1416(\frac{22}{7}), \ \times \frac{1}{6} \times d$, or $d^3 \times .5236(\frac{11}{21})$, or $\frac{1}{6} \pi d^3$.

Or, area *gt cir* $= r^2 \times 3.1416(\frac{22}{7})$, or πr^2.

Surf sph $= r^2 \times 3.1416(\frac{22}{7}) \times 4$, or $4 \pi r^2$.

Cont sph $= r^2 \times 3.1416(\frac{22}{7}) \times 4 \times \frac{1}{3} \times r$, or $\frac{88}{21} \times r^3$, or $\frac{4}{3} \pi r^3$.

Contents $\div \frac{4}{3} \pi (\frac{88}{21}) = r^3$. Contents $\div \frac{1}{6} \pi (.5236$ or $\frac{11}{21}) = d^3$.

METRIC SYSTEM IN A NUTSHELL.

The unit, meter, is one ten-millionth of the distance from the equator to a pole.

$\frac{25000 \times 320 \times 16\frac{1}{2} \times 12}{4 \times 10000000} = 39.37$ in. (39.6), or 1 meter, $\frac{1}{10000000}$ of $\frac{1}{4}$ of a meridian circle.

PREFIXES.		UNITS.
Myria	= 10000.	Meter, — unit of Linear Measure.
Kilo	= 1000.	Are, — unit of Surface Measure.
Hecto	= 100.	Stere, — unit of Solid Measure.
Deka	= 10.	Liter, — unit of Liquid Measure.
Deci	= .1	Gram, — unit of Weight.
Centi	= .01	
Milli	= .001	

1 sq. dkm. = 1 are (a.).

$$\frac{393.7^2\,(154999.69)}{144 \times 272\tfrac{1}{4} \times 160} = .024 + \text{ acres.} \quad \text{Ha.} = 2.4 \text{ acres.}$$

1 cm. = 1 stere (st.).

$$\frac{39.37^3\,(61023.377)}{1728 \times 27, \text{ or } 128} = 1.307 \text{ cu. yd., or } .276 \text{ cd.}$$

1 c.dm. = 1 liter (l.).

$$\frac{3.937^3\,(61.023377)}{231 \text{ (cu. in. in 1 gal.)}} = .264 \text{ gal., or } 1.057 \text{ qt.}$$

1 c.cm. = 1 gram (g.).

$$\frac{.3937^3\,(.061023377) \times 62.5 \times 7000}{1728} = 15.4 \text{ grains.}$$

(62.5 lb. in 1 cu. ft. water; 7000 gr. in 1 lb.)

Ratio for length is 10.

sq. or q.dk. = 10^2 = 100.
qh. = 100^2 = 10000.
qk. = 1000^2 = 1000000.

Ratio for squares is 100.

c.dk. = 10^3 = 1000.
ch. = 100^3 = 1000000.
ck. = 1000^3 = 1000000000.

Ratio for cubic units is 1000.

This key unlocks all the work of the metric system. **Turn it.**

TOPICAL OUTLINE.

Illustrate, whenever possible.

The decimal point, — the base from which values of numbers are determined.

Three orders, same for all periods. First five periods.

Separating point. How write, how read, numbers?

Scale of reduction and reduction; as, $3\,h\ 2\,t\ 7\,u = 300\,u + 20\,u + 7\,u = 327$ units; 3 yd. 2 ft. 7 in. = 108 in. + 24 in. + 7 in. = 139 inches. Uniform scale; varying scale.

Addition: How add? Analysis of addition.

Subtraction: Terms of, — How subtract? Analysis of the operation.

Formulas with m, s, and r. $r = m - s$, $10 = 15 - 5$; etc.

Multiplication: Terms of, — How multiply? Analysis of operation.

Names of results. 375 bu. at $2, 2 times 375 = 750, or $750.

Proofs. Formulas, using *pd*, *mr*, and *md*.

Effect of multiplying or dividing either or both *md* and *mr*.

Where use these principles?

$6 \times 3 = 18 \div 9 = 2$, not true; show it.

Division: Terms of, — How divide? Analysis of operation. Proofs.

Formulas, using *dd*, *dr*, and *quo*. $2)3^{1}7^{1}4(187$, *bad*.

Effect of multiplying or dividing either or both *dd* and *dr*. Six cases.

Where use these principles?

PROPERTIES OF NUMBERS.

Divisor or factor.

Prime, composite, odd, and even numbers. Numbers prime to each other.

Divisors 2, 3, 5, and 9, — when and why?

Divisor of two or more numbers, a divisor of their sum and difference.

Common factor. Greatest common factor.

G. c. f. by inspection, — three cases. G. c. f. must contain what of the numbers? never can be larger than what?

How find the g. c. f. of large numbers? Reason for method. How know that you have the g. c. f.? Where use g. c. f.?

Multiple or dividend. Common multiple. L. c. m.

L. c. m. must contain what of the numbers? never can be smaller than what?

Find l. c. m. by inspection, — three cases.

Reason for the ordinary method of finding l. c. m.

Where use l. c. m.?

Cancellation: Principle of, — Where used?

Fractions: Fractional unit. Unit of a fraction. Value of a fraction.

Fractions like simple division.

Principles of division applied to fractions.

Like fractions.

Reduction of fractions: To lowest terms. When in lowest terms? Principle.

The l. c. denom., object of finding it. Principle.

To integers or mixed numbers.

Mixed numbers to fractions; $7\frac{3}{4} \times 4 = 31$, $31 \div 4 = \frac{31}{4}$; explain.

Addition of fractions; of two fractional units; of mixed numbers.

Subtraction of fractions; of two fractional units; of mixed numbers, — three cases.

Multiplication of fractions.

A fraction by an integer; $\frac{4}{5} \times 3 = \frac{12}{5}$. Why? $\frac{5}{9} \times 3 = \frac{5}{3}$. Why?

$8 \times \frac{5}{6} = \frac{8 \times 5}{6}$; $8 \times 5 = 40$; $40 \div 6 = \frac{40}{6}$, etc. Explain.

A fraction by a fraction. Explain.

A mixed number by an integer; an integer by a mixed number; a mixed number by a mixed number.

Compound fractions.

Division of fractions.

A fraction by an integer; $\frac{8}{9} \div 4 = \frac{2}{9}$. Proof. $\frac{7}{8} \div 4 = \frac{7}{32}$. Proof.

An integer by a fraction; $7 \div \frac{3}{4} = (7 \div 3) \times 4 = \frac{28}{3}$. Explain.

A fraction by a fraction; $\frac{7}{8} \div \frac{5}{7} = \frac{7}{8} \times \frac{7}{5} = \frac{49}{40}$. Why?

Mixed number by a whole number; $5)\,72\frac{3}{4}\,(14\frac{11}{20}$. Analyze.

Whole number by a mixed number;
$$\begin{array}{rl} 5\frac{1}{2}) & 72 \\ \underline{2} & \underline{\ \ 2} \\ 11) & 144\,(13\frac{1}{11}. \end{array}$$
Explain.

Mixed number by a mixed number;
$$\begin{array}{rl} 5\frac{1}{2}) & 72\frac{3}{4} \\ 4 & \ \ 4 \\ 22) & \overline{291}\,\overline{(13\frac{5}{22}}. \end{array}$$
What principle?

Complex fraction; $\frac{7\frac{1}{2} \times 4}{9\frac{3}{4} \times 4} = \frac{30}{39} = \frac{10}{13}$. What principles?

Another way by which the last two expressions may be solved.

Relation of numbers.

The number compared; the number with which it is compared, or the base.

Kind of numbers that may be compared.

Make three questions out of one. Reduce each to the form of an equation. What is an equation? Apply the formulas of multiplication.

Make relation of number questions with integers, with fractions, with decimals, with compound numbers, and with the per cent sign (%).

(Pupils should do as asked and state *exactly* what they do. Work in arithmetic *is* or it *is not*.)

Analysis in fractions.

What is a "base unit"? Make, analyze, and verify several "A, B, and C" questions.

Decimals.

What is a decimal? How compare with a whole number?

Use of decimal point.

How write, how read, a decimal?

Reduction to fractions and from fractions. Principles.

How add, how subtract, decimals?

How point off product? Reason.

How point off the quotient? Reason. Much drill.

Contractions. Aliquot parts.

Multiply by $16\frac{2}{3}$, etc. 11, 12, 21, 248, 98, $4\frac{1}{2}$, and $4\frac{1}{2}$, $4\frac{3}{7}$, and $4\frac{4}{7}$.

Principles for each.

Divide by easy parts of 100. Principles.

Concrete numbers. **Denominate numbers.**

Compound denominate numbers.

Reduction of. How differ from simple numbers?

Reduce feet to inches; how, with reasons.

Reduce inches to feet; how, with reasons.

Change from higher to lower denomination; from lower to higher.

Relation of compound denominate numbers.

Show that addition, subtraction, multiplication, and division of compound denominate numbers differ from like operations with simple numbers, only in scale used.

Why so much time given to compound denominate numbers?

Measure distances; find surfaces and solids. Make *real.*

Carpeting. How carpets are made? Matching.

Papering. How done? Matching.

Plastering. How estimated? Deductions.

Masonry. What measurements? Number of bricks, how found? Why divide by $16\frac{1}{2}$ to find the number of perches?

Board measure. Unit of. Number of units, how estimated? Tapering board.

Longitude and time.

What is longitude? What is time?

Explain the scale number, 15. Prime meridian.

Make map or single diagram to show difference in longitude.

Compare time of any place with that of a place east or west of it.

Think. See sun rise. Faster time. Slower time.

Reduce, mentally, difference of longitude to difference of time and the reverse.

Metric system.

The base unit; meaning of the word; value, how found? Find it.

Name and give value of prefixes. Name the different units and the corresponding measure. Show the value of each as compared with the base unit.

Find the value of each.

Explain the scale of reduction in each, linear, surface, and solid measure.

How reduce English units to metric? Your weight in metric units.

General analysis.

What is the starting-point? What is the plan of work?

Proportion.

Terms: how many, names, couplets? Develop the signs.

Relation of second term to first compared with that of fourth to third.

General principle.

Simple proportion; compound.

How arrange the question for proportion? How state? How solve?

"More or less" question.

Partnership.

What is it? Partners, firm, capital, dividend.

How work? General principle.

Percentage.

Per cent, %, hundredths, $\frac{1}{100}$.

Analyze the reduction of % to a decimal, and a decimal part to %.

Terms: compared with those of multiplication.

Compare questions with those of relation of numbers. From one question make three, and for each, an equation, and both percentage and multiplication formulas.

Profit and loss.

What is profit? On what gain or lose? When lose?

p or l is p % or l % of c. Explain.

Reduce to simple percentage question. Make formulas for each term.

Commission.

What is commission? On what paid? How estimated?

Terms: make questions and formulas for each.

Insurance.

Explain the business. What runs it? Parties interested. Kinds.

Terms: make questions and formulas for each.

Taxes.

For what paid? by whom? to whom? What offices connected with the business? Real estate. Personal property. Poll. Tax is tax % of assessed value. Develop.

Duties. For what paid? by whom? to whom? where?

Revenue. Protective tariff. *Ad valorem.* Specific.

Duty is duty % of value. Develop.

Stocks. Corporation. Capital. Shares. Par value. Market value. Dividend. Brokerage. Dividend and brokerage reckoned upon what?

Show that a 6 % dividend may not be a 6 % investment.

Make and solve a question in stocks.

Interest. Business of. Terms of. Difference in time.

Interest is interest % $(r \times t)$ of principal. Develop. Six per cent method; develop.

Other per cents. Accurate interest. Make formulas for different terms (problems).

pw (principal) = debt (amount) ÷ (1 + *r* affected by time) amount of $1. Explain.

Partial payments.

Make note with indorsements. Solve it.

Simple interest repeated. Payments deducted. Show it.

Compound interest.

Annually. Semi-annually.

Show that it is simple interest repeated regularly.

Bank discount.

Bank. Discount. Proceeds. Term of discount.

Interest-bearing note. Non-interest-bearing note.

For what time does a bank take interest? Operation, only simple interest.

Commercial discount.

Who make it? Why? What is 10% of? What question of percentage represented?

Exchange.

Parties interested. Terms used. Bills of. Sight draft. Time draft.

Acceptance. Time discount, who receives it? Rate of exchange.

How find price of $1 exchange, in cash? Price, quantity, and cost questions.

Average of payments. Payments. Average. Explain.

Average.

Summary of cost ÷ summary of quantity, etc. Explain.

Powers and roots.

Root, exponent, radical sign, power, index.

(Exponent does not show how many times a number is multiplied by itself. Show it.)

Number of figures in the root compared with those in the second power; in the third. In square root, how point off the power? Reason.

Why take two times the root, regarded as tens? Explain figure.

In cube root how point off the power? Explain.

Why take three times the square of the root already found, etc.?

What do the divisors and their parts represent? What the remainders?

Give analysis of cube root, using drawings and blocks.

Use of cube root. Of square root.

Mensuration.

Make and define the different kinds of lines, angles, surfaces, and solids.

Explain methods for finding the different surfaces.

Explain methods for finding contents of different solids, so far as possible.

Make a distinction between to *believe* and to *know*.

Finally. Reviews and frequent reviews are necessary to develop and fix principles.

Memorized knowledge of arithmetic is not arithmetical knowledge.

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